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CARACTERISATION ET ETALONNAGE DES SYSTEMES D'IMAGERIE DE STOKES A MATRICE DE FILTRES POLARISEURS

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I, Yilbert GIMÉNEZ, declare that this thesis titled, “Caractérisation et étalonnage des systèmes d’imagerie de Stokes à matrice de filtres polariseurs” and the work presented in it are my own. I confirm that:

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- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

“There ought to be something very special about the boundary conditions of the universe. And what can be more special than that there is no boundary, and there should be no boundary to human endeavor. We are all different. However bad life may seem, there is always something you can do, and succeed at.

While there is life, there is hope”

Stephen Hawking

UNIVERSITÉ DE HAUTE-ALSACE

Abstract

Équipe Fonctions Optiques et Traitement de l'Information
Institut De Recherche En Informatique, Mathématiques, Automatique Et Signal

Doctor of Philosophy

Caractérisation et étalonnage des systèmes d'imagerie de Stokes à matrice de filtres polariseurs

by Yilbert GIMÉNEZ

La polarisation est l'une des propriétés fondamentales de la lumière, les autres étant l'intensité, la fréquence et la cohérence. De nos jours, l'utilisation de la polarisation de la lumière connaît un intérêt croissant à mesure que les instruments permettant son analyse deviennent disponibles sur le marché. Cette thèse se concentre sur l'étalonnage des systèmes d'imagerie polarimétrique à base de capteurs à division du plan focal. Ces capteurs ne nécessitent qu'une seule prise d'image, d'où leur compacité et leur efficacité. La principale contribution de la thèse est une procédure expérimentale d'étalonnage simplifiée, qui réduit significativement la complexité des méthodes précédentes.

Polarization is one of the fundamental properties of light, the others being intensity, frequency and coherence. Nowadays, the use of the polarization of light knows an increasing interest as the instruments allowing its analysis become available on the market. This thesis focuses on the calibration of polarimetric imaging systems based on focal plane division sensors. These sensors only require one image capture, hence their compactness and efficiency. The main contribution of the thesis is a simplified experimental calibration procedure, which significantly reduces the complexity of the previous methods.

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List of Abbreviations

A	Analyzer vector of a polarization element
AWGN	Additive W hite G aussian N oise
B	Magnetic field vector with scalar components B_x and B_y
CN	Condition N umber
CPFA	Color P olarization F ilter A rray
d	D istance [m]
D	D iattenuation
DOP	D egree O f P olarization
DOLP	D egree O f L inear P olarization
DoA	D ivision o f A perture
DoAm	D ivision o f A mplitude
DoFP	D ivision o f F ocal P lane
DoT	D ivision o f T ime
DSNU	D ark S ignal N on U niformity
E	Electric field vector with scalar components E_x and E_y
EFL	E ffective F ocal L ength
ER	E xtinction R atio
EWV	E qually W eighted V ariance
FPN	F ixed P attern N oise
IFOV	I ntermediate F ield O f V iew
I	I ntensity vector of a polarimeter
I_0	Light I ntensity polarized along the horizontal axis x
I_{45}	Light I ntensity polarized at 45°
I_{90}	Light I ntensity polarized along the vertical axis y
I_{135}	Light I ntensity polarized at -45°
I_l	Light I ntensity polarized in left circular state
I_r	Light I ntensity polarized in right circular state
k	Spatial frequency (also known as wavenumber) [cycles/d]
M	M ueller Matrix of a polarization element
n	Refractive index (also known as refraction or refringent index) of a material
PFA	P olarization F ilter A rray
PRNU	P hoto R esponse N on U niformity
PSA	P olarization S tate A alyzer
PSG	P olarization S tate G enerator
PSN	P oisson S hot N oise

PSNR	P eak S ignal to N oise R atio
Q	Number of polarization state analyzers in a polarimeter
RMSE	R oot M ean S quare E rror
$\mathbf{S}_{in}$	I ncident S tokes Vector
$\mathbf{S}_{out}$	O utgoing S tokes Vector
S_0	First component of the S tokes Vector
S_1	Second component of the S tokes Vector
S_2	Third component of the S tokes Vector
S_3	Fourth component of the S tokes Vector
T_{max}, q	M aximum T ransmission of a polarization element (Co-polarized)
T_{min}, r	M inimum T ransmission of a polarization element (Cross-polarized)
$\mathbf{W}$	Analyzer matrix of a polarimeter
ϕ	Phase shift between the electric field components E_x and E_y
ψ	A ngle O f P olarization (AOP)
ε	E llipticity O f P olarization (EOP)
λ	Wavelength of an electromagnetic wave
δ	Retardance or phase change introduced by a polarization element
θ	Orientation axis of a polarization element
ξ	Numerical nonideality error of in the analysis vector components
σ	Temporal standard deviation of the AWGN
η	Quantum Efficiency of the photosensor
Γ	Autocovariance matrix operator applied to the vector put as superscripts
ω	Angular frequency [rad/s]

*Dedicado a la memoria de mi padre Simón Enríque y de mi abuela
Otilia Pinto. ...*

Introduction

Polarization of light is one of its fundamental properties, the others being its intensity, frequency and coherence. Reptiles or birds, for example, are capable of perceiving polarized information, whereas humans, like any other mammals, cannot. They can only perceive polarization through the use of extra materials or assemblies suitable to sense the wave nature of the electromagnetic radiation.

The history of polarization discovery is well described in a recent book by Goldstein [1]. The investigation of polarized light began to be documented by Erasmus Bartholinus (1625-1698) with the discovery of the phenomenon of double refraction in calcite crystal. Later, Christian Huygens (1629-1695), interpreted double refraction in the same material, followed by the work of Isaac Newton (1642-1724) who interpreted the phenomenon considering light waves as longitudinal, that is, similar to sound waves. Several years later, Sir David Brewster (1781-1868) discovered that at a particular angle of incidence the reflected light viewed through a calcite crystal could be extinguished, and further investigations took him to the conclusion that there was a relation between that was to be called Brewster's angle and the refractive index of the glass. The wave theory was placed on a theoretical foundation by Augustin Fresnel (1788-1827) using the Fresnel-Huygens integral to solve the problem of diffraction. The electrodynamics theory proposed in 1876 by James Clerk Maxwell (1831-1879) brings the final confirmation of the truth of the wave theory of light. In 1852, Sir George Gabriel Stokes established a mathematical description of incoherent or partially polarized light [2]. Then in 1890, Henri Poincaré (1854-1912) used a complex plan to represent the state of polarization and later, he proposed a spherical representation to describe these states of polarization which is still very much in use today. Subsequently, two matrix calculation methods were proposed to study the polarization phenomenon, namely, Jones calculus, which uses a 2-by-2 matrix of complex values to describe the evolution of fully polarized light [3]. The second method is the Mueller calculus [4] based on a 4-by-4 matrix of real values that can be seen as a transfer function of the Stokes vector (a 4 component vector of real values containing all polarization information), and it is used to describe the alteration of polarization characteristics by a material element.

Nowadays, the use of polarization of light is widely exploited. For example, many of our personal computers, tablets, and phones feature liquid crystal screens, where the electrical modulation of liquid crystals, and thus the polarization state of light, permit to vary the light intensity. As for entertainment, polarization is used as a cutting edge technology in 3D films, allowing to perceive a different image for each eye and therefore to create a stereoscopic 3D effect. In scientific research, polarimetric imaging systems are used in fields such as astronomy, remote sensing, medical diagnostics, industrial control,

and military defense, among others. Different technological architectures for controlling and imaging the polarization of light have emerged. As for every optoelectronic system, polarimetric imaging systems are susceptible to different sources of inaccuracies and noise that must be quantified and/or mitigated, to prevent any imaging polarimetry application from failing. This work aims at the evaluation of image processing techniques dedicated to polarimetric characterization and calibration, along with the optimization of the calibration procedure for monochrome polarization cameras.

This thesis has been carried out in the Optical Functions and Information Processing (FOTI) team, from the Research Institute in Computer Science, Mathematics, Automation and Signal (IRIMAS, formerly MIPS) in Université de Haute-Alsace (UHA). For the past twenty years, IRIMAS has developed substantial expertise in the study of imaging polarimetry with the use of division-of-time liquid crystal modulators [5, 6, 7]. Nevertheless, this thesis focuses on the division of focal plane sensors, which offer the snapshot capability, and where the compactness and efficiency is increased.

Contribution

The major contribution of this work is about Division-of-Focal-Plane Stokes imagers.

- On the one hand, a state-of-the-art of characterization and calibration techniques is provided. Then, from the calibration point of view, an optimum algorithm is established as well as a simplification of the procedure required to perform the calibration procedure.
- On the other hand, a study is carried out to investigate the influence of the quality of the imaging optics (namely its f-number) on the precision of estimation of polarimetric parameters.

Structure of the manuscript

The manuscript is made up of 4 chapters. The first chapter contains the mathematical formalism used to represent the polarization of light, i.e., the Stokes formalism, the Poincaré sphere, and the Mueller formalism. The end of the chapter will present the general concept of polarimetry.

The second chapter outlines the different architectures of polarimetric imaging. The Stokes and Mueller imager architectures are presented and compared. Then, the mathematical formulation of the pixel model is given for the specific case of the polarization filter array imager. Finally, a few existing commercially available devices are presented.

The third chapter deals with the calibration algorithms dedicated to polarization filter array imager, which aims to calibrate the nonidealities of the polarimeter using a training procedure. Four existing calibration techniques are implemented, and a benchmark is proposed. The goal of the evaluation is first to compare which algorithm performs better

with regard to polarization error, and then to investigate both the influence of the dynamic range and number of polarization angle stimuli of the training data.

The fourth chapter is intended to study the impact of the optical path due to the use of objective lenses in the polarimetric degradation in respect with two different assembly structures, as well as the impact of nonuniformity of light in the characterization of the polarizing properties.

Finally, the conclusion is presented, consisting of a summary of the main the output of each chapter and how they are related to one another, which turns out in a simplified practical calibration procedure which reduces the complexity of previous methods. Moreover, some perspectives on the future of polarization imaging and possible future work in this topic is stated.

Chapter 1

Polarization of light

This chapter consists mainly of a synthesis of the different mathematical formulations adapted to the study of polarization of light and to polarimetric imaging systems. As different topics are developed, the definitions of the vocabulary of polarization-altering devices related to the phenomenon in question are promptly presented.

1.1 Polarization state of an electromagnetic wave

A plane electromagnetic wave is a spatio-temporal entity that can be described by an electric field vector $\mathbf{E}$ and magnetic field vector $\mathbf{B}$, orthogonal in respect to one another and, in turn, both orthogonal to the direction of propagation. Since both fields are time-dependent, then they can be considered together as a coupled electromagnetic field according to Maxwell's equations.

The vector nature of light phenomena must be taken into account in the description of polarization. The direction of oscillation of the electric field is called polarization. By convention, only the electric field vector, denoted with bold notation, $\mathbf{E}$ is considered to describe the polarization direction. The study of polarization is done on the projections of the electric field vector $\mathbf{E}$ on an orthogonal base perpendicularly to the direction of propagation. Figure 1.1 depicts the projections of electric field vector $\mathbf{E}$ in the plane xy . Considering a point in the axis of direction of propagation z , in an instant t , the electric field can be written in 2-tuple notation as [8]:

$$\mathbf{E} = (E_x(z, t), E_y(z, t)) . \quad (1.1)$$

Where E_x and E_y represent the scalar components in the axis indicated in subscript, z the propagation axis and t the time. In a homogeneous medium without dispersion and losses, these projections, also called polarization components, can be written in column vector notation as:

$$\mathbf{E} = \begin{bmatrix} E_x(z, t) \\ E_y(z, t) \\ E_z(t) \end{bmatrix} = \begin{bmatrix} E_{0x} \cos(\omega t - kz + \phi_x) \\ E_{0y} \cos(\omega t - kz + \phi_y) \\ 0 \end{bmatrix} . \quad (1.2)$$

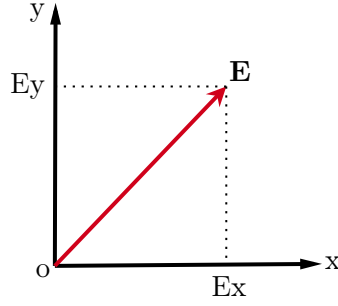


FIGURE 1.1: The electric field vector, denoted with bold notation $\mathbf{E}$, is made up with the scalar projections (E_x, E_y) whose values oscillate over time, giving the total electric field a specific trajectory. The origin of the plane xy is the crosssection of the propagation axis, z which comes out of the paper.

Where E_{0x} and E_{0y} represent the maximum amplitude of each component, ϕ represent the phase shift between the two components, ω is the angular frequency, k is the wavenumber (spatial frequency); which is directly related to the wavelength λ by $k = \frac{2\pi}{\lambda}n$, where n is the refractive index of the propagation medium. The electric field vector components can be depicted in space as shown in Figure 1.2.

By using $\phi = \phi_y - \phi_x$, Equation 1.2 can be simplified as:

$$\mathbf{E} = \begin{bmatrix} E_x(z, t) \\ E_y(z, t) \\ E_z(t) \end{bmatrix} = \begin{bmatrix} E_{0x} \cos(\omega t - kz) \\ E_{0y} \cos(\omega t - kz + \phi) \\ 0 \end{bmatrix}. \quad (1.3)$$

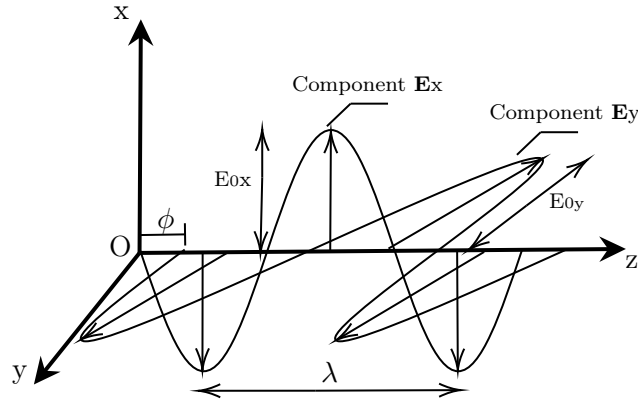


FIGURE 1.2: Propagation diagram of the electric fields components of an electromagnetic wave.

The polarization state of an electromagnetic wave is defined by the curve described by the tip of the electric field vector $\mathbf{E}$ as a function of time, projected into the plane orthogonal to the direction of propagation. By considering z as constant and equalizing the scalars projections in Equation 1.3 one gets:

$$\begin{cases} E_x(t) = E_{0x} \cos(\omega t) , \\ E_y(t) = E_{0y} \cos(\omega t + \phi) . \end{cases} \quad (1.4)$$

The two identities in Equation 1.4 can be combined by eliminating the time dependence of the term ωt between the polarization components E_x and E_y , in a way that one gets a representation of the trajectory of the electric field vector's tip projected in the plane xy as follows:

$$\left(\frac{E_x(t)}{E_{0x}}\right)^2 + \left(\frac{E_y(t)}{E_{0y}}\right)^2 - 2\frac{E_x(t)E_y(t)}{E_{0x}E_{0y}}\cos(\phi) = \sin^2(\phi) . \quad (1.5)$$

If ϕ is not constant over time, then the wave is said to be unpolarized and the locus of points described by the tip of the optical electric field vector as it propagates obeys a random trajectory. If ϕ is constant over time, Equation 1.5 is recognized as the equation of an ellipse, and it shows that at any instant of time the said locus describes an elliptical shape. This behavior is referred to as optical polarization [1] and the equation ruling the behavior is called polarization ellipse as shown in Figure 1.3. Elliptical polarization corresponds to the most general case of a totally polarized wave, that is, when the electric field trajectory is deterministic.

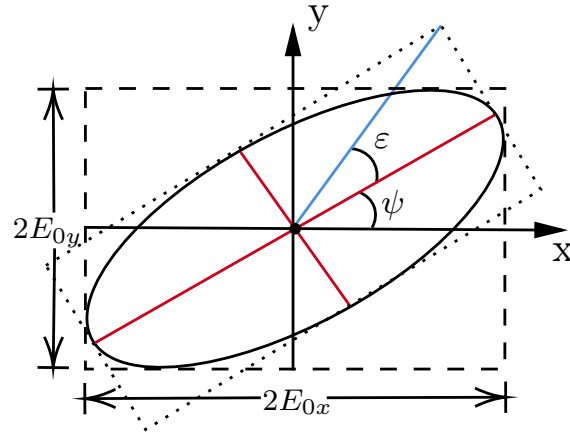


FIGURE 1.3: Polarization ellipse. The origin of the plane xy is the cross-section of the propagation axis z which comes out of the paper.

The handedness of the polarization ellipse as it revolves around the propagation axis (describing a helix in 3D space as depicted in Figure 1.4); is linked to the parameter ϕ , and by convention, it is determined for an observer as if looking in the direction of the light source. While $\sin(\phi) > 0$, the ellipse is traversed clockwise and the polarization is said to be right-handed. Accordingly, if $\sin(\phi) < 0$, the ellipse is traversed counterclockwise and the polarization is said to be left-handed. Two particular cases of elliptical polarization are worthy to mention: when $\sin(\phi) = 0$, the trajectory of the tip of the electric field vector oscillates along a straight line, and light is said to be linearly polarized (sometimes called plane polarized). When $\sin(\phi) = \pm 1$, the figure described by the electric field is a circle and the polarization is said to be left circular if $\sin(\phi) = -1$, and right circular if $\sin(\phi) = +1$.

In addition, the ellipse is characterized by two geometrical parameters: the angle of orientation of the ellipse ψ in respect to the x axis and the ellipticity¹ ϵ in respect to the

¹The parameter ellipticity quantifies the ellipse's geometrical compression or flattening.

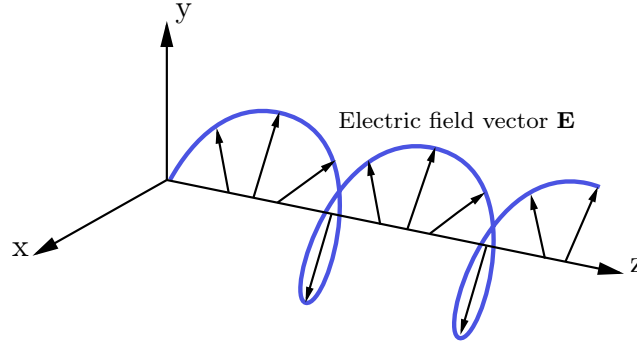


FIGURE 1.4: Polarization helix. The handedness convention of the propagating electric field vector as viewed from the receiver is determined by the parameter ϕ and related to the direction of propagation.

major axis. These two quantities are expressed as a function of the electric field vector components and the parameter ϕ as follows:

$$\tan(2\psi) = 2 \frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2} \cos(\phi) , \quad (1.6)$$

$$\tan(2\varepsilon) = 2 \frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2} \sin(\phi) . \quad (1.7)$$

Several mathematical formulations can be used to describe the polarization of light and its interactions with matter, depending on whether the light is totally or partially polarized. The Jones formalism allows describing the polarization when light is totally polarized, but as in the majority of cases of real measurements, the light is partially polarized, and therefore, the Jones vector will not be detailed here. This manuscript is focused on a convenient alternative to the more common description of incoherent or partially polarized radiation, namely, the Stokes vector that is detailed down below.

1.2 Stokes parameters

The Stokes parameters are a set of values that describe the polarization state of electromagnetic radiation [2]. The four Stokes parameters can be arranged in a column matrix called the Stokes vector. This vector can be represented in a mathematical four-dimensional space that can be called polarization space. Each component represents a value which conveys a piece of information as follows:

- S_0 associated to the total illumination intensity, which is not a polarimetric property of light.
- S_1 associated to the linear polarization relative to the vertical and horizontal components.
- S_2 associated to the linear polarization relative to the 45° and -45° components.
- S_3 associated to the circular polarization component.

In several cases, the circular contribution of polarized light reflected by materials can be neglected, particularly when capturing outdoor scenes illuminated by rather unpolarized natural sources such as the sunlight. The scattering of light is mostly linearly partially polarized, i.e. the principal polarization state is linear. In the said case, the S_3 component can be considered null. However, when studying man-made objects, made of birefringent materials, this assumption may not be fulfilled.

Equation 1.5 describes the elliptic trajectory of the electric field vector, and it is valid only at a given instant of time. While E_{0x} , E_{0y} and ϕ are constant, $E_x(t)$ and $E_y(t)$ continue to be time-dependent. In order to represent polarization ellipse in terms of the observable optical field, a representation over the time-averaged intensity can be carried out. Because this period of time is much longer with respect to the time for a single oscillation, it can be taken to be infinite. However, in view of periodicity of $E_x(t)$ and $E_y(t)$, we need to average Equation 1.5 over a single period of oscillation. The time average is represented by the symbol $\langle \dots \rangle$, so that the polarization ellipse can be written as:

$$\frac{\langle E_x(t)^2 \rangle}{E_{0x}^2} + \frac{\langle E_y(t)^2 \rangle}{E_{0y}^2} - 2 \frac{\langle E_x(t)E_y(t) \rangle}{E_{0x}E_{0y}} \cos(\phi) = \sin^2(\phi) . \quad (1.8)$$

The average value of the three terms of Equation 1.8 are:

$$\langle E_x^2(t) \rangle = \frac{1}{2} E_{0x}^2 , \quad (1.9)$$

$$\langle E_y^2(t) \rangle = \frac{1}{2} E_{0y}^2 , \quad (1.10)$$

$$\langle E_x(t)E_y(t) \rangle = \frac{1}{2} E_{0x}E_{0y} \cos(\phi) . \quad (1.11)$$

By substituting Equations 1.9, 1.10 and 1.11 in Equation 1.5 one gets:

$$2E_{0x}^2 E_{0y}^2 + 2E_{0x}^2 E_{0y}^2 - (2E_{0x}E_{0y} \cos(\phi))^2 = (2E_{0x}E_{0y} \sin(\phi))^2 . \quad (1.12)$$

By completing the square, Equation 1.12 can be arranged as:

$$(E_{0x}^2 + E_{0y}^2)^2 - (E_{0x}^2 - E_{0y}^2)^2 - (2E_{0x}E_{0y} \cos(\phi))^2 = (2E_{0x}E_{0y} \sin(\phi))^2 . \quad (1.13)$$

From Equation 1.13, the Stokes parameters of a plane wave are commonly defined as follows:

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y} \cos(\phi) \\ 2E_{0x}E_{0y} \sin(\phi) \end{bmatrix} . \quad (1.14)$$

Equation 1.14 is also called the Stokes vector equation for a plane wave. It is also possible to estimate the four Stokes vector components from several intensity measurements

as follows:

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \propto \begin{bmatrix} I_0 + I_{90} \\ I_0 - I_{90} \\ I_{45} - I_{135} \\ I_l - I_r \end{bmatrix} . \quad (1.15)$$

Where I_0 and I_{90} represent the light intensities polarized along the horizontal and vertical axis x and y respectively, I_{45} and I_{135} represent the light intensities polarized at 45° and -45° , and I_l and I_r represent the light intensities polarized in a left circular and right circular state.

The Stokes vector is often normalized with respect to the S_0 component to represent the state of polarization of light independently of its intensity, in a way that, the last three vector components are included in the interval $[-1, 1]$, such that:

$$\mathbf{S} = \begin{bmatrix} 1 \\ S_1/S_0 \\ S_2/S_0 \\ S_3/S_0 \end{bmatrix} . \quad (1.16)$$

As a result, remarkable polarization states can be expressed for fully linearly polarized light as follows:

$$\mathbf{S}_{\text{LHP}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{S}_{\text{L}+45} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{S}_{\text{LVP}} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{S}_{\text{L}-45} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} . \quad (1.17)$$

Where, $\mathbf{S}_{\text{LHP}}$ represents linear horizontally polarized light, $\mathbf{S}_{\text{L}+45}$ the $+45^\circ$ polarized light, $\mathbf{S}_{\text{LVP}}$ the vertically polarized light, and $\mathbf{S}_{\text{L}-45}$ the 135° polarized light. Also, the left circular $\mathbf{S}_{\text{LCP}}$ light, and right circular polarized light $\mathbf{S}_{\text{RCP}}$ can be expressed as follows:

$$\mathbf{S}_{\text{LCP}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{S}_{\text{RCP}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} . \quad (1.18)$$

In the case of fully polarized light, the relation among the Stokes vector parameters obeys the identity:

$$S_0^2 = S_1^2 + S_2^2 + S_3^2 . \quad (1.19)$$

In the case of unpolarized light, all the parameters equal zero, except for S_0 which satisfies the inequation:

$$S_0 > 0 . \quad (1.20)$$

When the light is partially polarized, the relation among the parameters is as follows:

$$S_0^2 \geq S_1^2 + S_2^2 + S_3^2 . \quad (1.21)$$

The Stokes vector allows defining the so-called polarimetric parameters, which are useful to characterize the polarization properties of light beams:

- Degree of polarization:

$$DOP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} . \quad (1.22)$$

The degree of polarization (DOP) is defined as the ratio of the polarized component state to the total intensity. The DOP varies between 0 for unpolarized light, and 1 for fully polarized light. Similarly, it can also define the degree of linear polarization (DOLP) by doing $S_3 = 0$ in the Equation 1.22.

We may also define the degree of polarization specific to circular polarization (DOCP) as:

$$DOCP = \frac{\sqrt{S_3^2}}{S_0} = \frac{|S_3|}{S_0} . \quad (1.23)$$

- Angle of polarization :

The angle of the principal state of polarization, henceforth angle of polarization (AOP), turns out to be the orientation angle ψ of the polarization ellipse, which describes the orientation of the polarization whether it is linear or elliptical. When the state of polarization is circular or fully unpolarized, the angle of polarization is undefined. Otherwise, it can be calculated as:

$$AOP = \psi = \frac{1}{2} \arctan \left(\frac{S_2}{S_1} \right) , \quad 0 \leq \psi < \pi . \quad (1.24)$$

By convention, the classical single-argument inverse tangent function is restricted such that $\arctan: \mathbb{R} \rightarrow]-\pi/2, \pi/2[$. As a result, to return a correct and unambiguous value for the angle ψ over the half-circle, as specified in Equation 1.24, the angle-of-polarization function must be redefined to satisfy AOP: $\mathbb{R} \rightarrow [0, \pi[$. To this effect, it is possible to define a 2-argument piecewise function which returns the halved argument (phase) of the complex number $S_1 + iS_2$, so that the signs of both Stokes components are used to determine the quadrant of the result. In other words [9]:

$$\psi(S_1, S_2) = \begin{cases} \frac{1}{2} \arctan \left(\frac{S_2}{S_1} \right) , & \text{if } S_1 > 0 , \\ \frac{\pi}{2} \text{sgn}(S_2) + \frac{1}{2} \arctan \left(\frac{S_2}{S_1} \right) , & \text{if } S_1 < 0 . \end{cases} \quad (1.25)$$

Where the sign function obeys: $\text{sgn}(x) = 1$ if $x \geq 0$, -1 otherwise. From a practical perspective, the performance of Equation 1.25 can be obtained by means of the 2-argument arctangent functions; available in common computer languages. The realization could slightly differ from one computer language to another, so the results might be inspected if a positive angular scale is desired.

Moreover, the ellipticity of polarization (EOP), ε is expressed as a function of the Stokes parameters as follows:

$$\sin(2\varepsilon) = \frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}}, \quad \frac{-\pi}{4} \leq \varepsilon < \frac{\pi}{4}. \quad (1.26)$$

The reduced Stokes vector can also be expressed as a function of the three above parameters as follows:

$$\mathbf{S} = DOP \begin{bmatrix} 1 \\ \cos(2\psi) \cos(2\varepsilon) \\ \sin(2\psi) \cos(2\varepsilon) \\ \sin(2\varepsilon) \end{bmatrix}. \quad (1.27)$$

In the case of linear polarization, Equation 1.27 is simplified as:

$$\mathbf{S} = DOLP \begin{bmatrix} 1 \\ \cos(2\psi) \\ \sin(2\psi) \\ 0 \end{bmatrix}. \quad (1.28)$$

1.3 Poincaré sphere

The Poincaré sphere is a graphic representation method of polarization states [10]. Each point on the surface of the sphere represents one polarization state as shown in Figure 1.5. The three reference axes are the Stokes vector components S_1 , S_2 and S_3 . The S_1 axis represents the horizontally and vertically linearly polarized components. The S_2 axis represents the linearly polarized components at 45° and -45° , and the S_3 axis represents the right and left circularly polarized components. In polar coordinates, a polarization state is characterized by the angle of polarization ψ and the ellipticity of polarization ε . The norm of the vector $\mathbf{S}$ represents the degree of polarization. The equator represents the locus of fully linearly polarized light, without any circular component. The poles represent respectively for the North Pole the state of right circular polarization, and for the South Pole the left circular polarization state.

Two cases are worthy to mention, on the one hand, fully polarized light (either circular or linear) will have a degree of polarization equal to one, and therefore, such a state will be represented by a point on the surface of the sphere. On the other hand, a partially or unpolarized wave will have a degree of polarization less than one, and consequently, those states will be represented by a point inside the sphere. In the case of fully unpolarized light,

the state will be represented by one point in the center of the sphere. Two diametrically opposite points correspond to orthogonal polarization states.

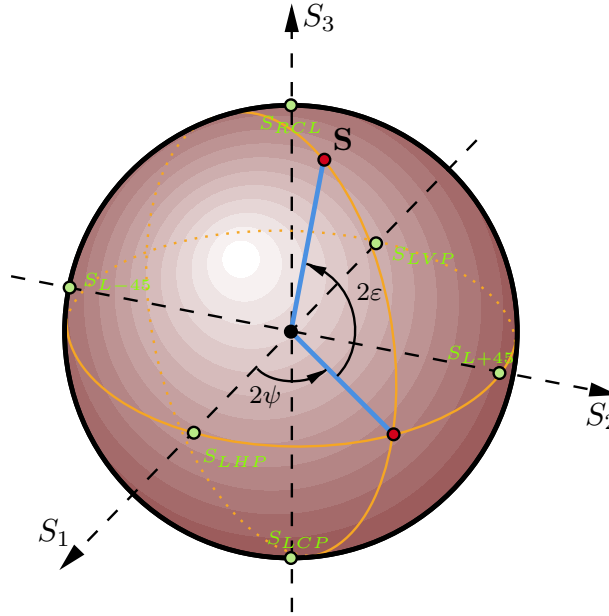


FIGURE 1.5: Poincaré sphere. Any polarization state can be represented by the spherical coordinate triplet (DOP, 2ψ , 2ε). The location of the six fully-polarized light states expressed in Equations 1.17 and 1.18 are depicted in green. All such states are located on the intersections of the surface of the sphere with the axes.

1.4 Mueller calculus

Mueller calculus was developed in 1943 by Hans Mueller. It consists of a matrix method for manipulating Stokes vectors. The Mueller matrix $\mathbf{M}$ is a 4×4 matrix with real-valued elements, which can be considered as the matrix transfer function of a polarization-altering device between an incoming and outgoing Stokes vectors. Thus, after a light interaction, an input Stokes vector $\mathbf{S}_{\text{in}}$ is transformed into an output Stokes vector $\mathbf{S}_{\text{out}}$ such as:

$$\mathbf{S}_{\text{out}} = \begin{bmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} = \mathbf{M} \mathbf{S}_{\text{in}} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}. \quad (1.29)$$

If the incoming light is characterized by a polarization state $\mathbf{S}_{\text{in}}$, passing through n cascaded polarization-altering devices whose Mueller matrices are $\mathbf{M}_i$, the outgoing polarization state $\mathbf{S}_{\text{out}}$ can be obtained as the sequential product of individual matrices in the order of the light passing through them [11]:

$$\mathbf{S}_{\text{out}} = \prod_{i=1}^n \mathbf{M}_i \mathbf{S}_{\text{in}}. \quad (1.30)$$

The Mueller matrix is an appropriate formalism for characterizing polarization measurements because it contains within its elements all the polarization properties and their form, either linear, circular, or elliptical. Polarization-altering devices (also called polarization elements) exhibit the following properties:

- Diattenuation and Extinction Ratio

The diattenuation D , also known as dichroism, is the property of an optical element whereby the intensity of the outgoing light depends on the polarization state of the incoming beam. The diattenuation is defined in terms of the maximum T_{max} and minimum T_{min} intensity transmittances as:

$$D = \frac{T_{max} - T_{min}}{T_{max} + T_{min}}. \quad (1.31)$$

$D \in [0, 1]$. $D = 1$ corresponds to a perfect diattenuation.

For a given Mueller Matrix $\mathbf{M}$. The intensity transmittance averaged over all incident polarization states (also known as mean transmittance) can be obtained from Equation 1.29 as $T_{ave} = m_{00}$. Likewise, the maximum and minimum transmittances can be obtained out of the elements of $\mathbf{M}$ as:

$$T_{max} = m_{00} + \sqrt{m_{01}^2 + m_{02}^2 + m_{03}^2}, \quad T_{min} = m_{00} - \sqrt{m_{01}^2 + m_{02}^2 + m_{03}^2}. \quad (1.32)$$

The polarizing figure of merit of an optical element is alternatively expressed in terms of the extinction ratio, ER as:

$$ER = \frac{T_{max}}{T_{min}} = \frac{1 + D}{1 - D}. \quad (1.33)$$

$ER \in [1, +\infty[$. $ER \rightarrow +\infty$ corresponds to a perfect diattenuation.

- Retardance

The retardance δ is the phase change an optical device introduces between the eigenpolarizations, i.e. polarization states that are transmitted unchanged except for a change in phase. This causes a phase shift between the components E_x and, E_y which is expressed as:

$$\delta = \frac{2\pi(n_1 - n_2)d}{\lambda}. \quad (1.34)$$

Where n_1 and n_2 are the refractive indices in the slow and fast axes respectively, d is the thickness of the plate and λ , the wavelength.

- Depolarization

It describes the capacity to transform incoming polarized light into depolarized light in the output beam.

When the Mueller matrix of an optical device is known, then the exiting polarization state can be estimated for an arbitrary incident polarization state. Different terms for polarization elements will be described hereafter.

1.4.1 Diattenuator

It consists of any homogeneous polarization element that exhibits anisotropic intensity attenuation, i.e. the intensity coefficient or transmittance, which depends on the state of polarization of the input electromagnetic beams [11]. Polarizers have a diattenuation close to one, but nearly all optical interfaces are weak diattenuators. Examples of diattenuators include: polarizers and dichroic materials, as well as metal and dielectric interfaces with reflection and transmission differences [12].

The Mueller matrix of an ideal polarizer whose orientation axis is defined by the azimuth in respect to the x axis, denoted by the angle $\theta \in [0, \pi[$, is as follows:

$$\mathbf{M}_{\text{ideal},\theta} = \frac{1}{2} \begin{bmatrix} 1 & \cos(2\theta) & \sin(2\theta) & 0 \\ \cos(2\theta) & \cos^2(2\theta) & \cos(2\theta)\sin(2\theta) & 0 \\ \sin(2\theta) & \cos(2\theta)\sin(2\theta) & \sin^2(2\theta) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (1.35)$$

For example, the Mueller matrix of an ideal linear polarizer horizontally oriented is as follows:

$$\mathbf{M}_{\text{ideal}_0} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (1.36)$$

Perfect polarizers obey an attenuation law named after the French physicist Étienne-Louis Malus who discovered it in 1809. Malus's law is illustrated in Figure 1.6. It indicates that an incoming polarization state with intensity I_{in} with an angle of polarization ψ , passing through a perfect linear polarizer with optical axis θ , produces an outgoing polarization state whose intensity I_{out} is equivalent to:

$$I_{out} = I_{in} \cos^2(\psi - \theta). \quad (1.37)$$

It is worthy to mention the case when fully unpolarized light passes through a θ -oriented polarizer, the angle of polarization ψ is undefined and the resulting light is fully polarized in the polarizer's direction; then by taking the average of $\cos^2(\theta)$ from Equation 1.37, we obtain that the light intensity is halved as:

$$I_{out} = \frac{1}{\pi} \int_0^\pi I_{in} \cos^2(\theta) d\theta = \frac{1}{2} I_{in}. \quad (1.38)$$

The same result expressed in Equation 1.38 can be obtained by using the Mueller matrix relation of Equation 1.29. For example, let us consider an ideal linear polarizer in

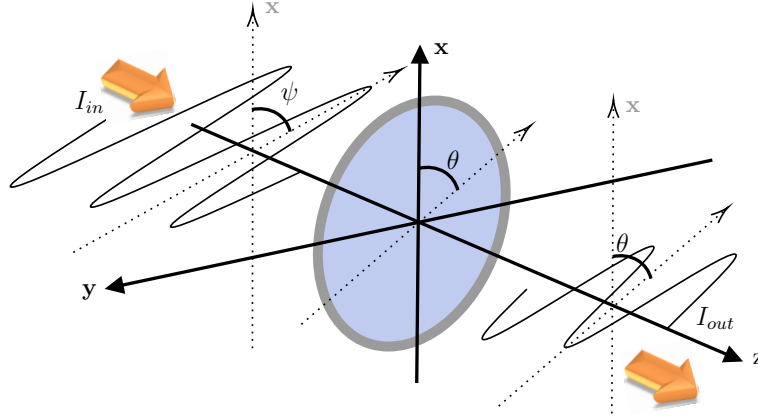


FIGURE 1.6: Illustration of Malus's law: the orientation axis of the polarizer forms an angle θ with the x axis. The polarized exiting wave follows the same direction as the axis of the polarizer but attenuated by the factor $\cos^2(\theta - \psi)$.

horizontal position characterized by the Mueller matrix pointed out in Equation 1.36 being traversed by an unpolarized beam, the output Stokes vector is defined as:

$$\mathbf{S}_{\text{out}} = \mathbf{M}_{\text{ideal}_0} \begin{bmatrix} S_{0_{in}} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} S_{0_{in}} \\ S_{0_{in}} \\ 0 \\ 0 \end{bmatrix}. \quad (1.39)$$

Using the identity expressed in Equation 1.15, the total exiting illumination intensity satisfies:

$$I_{\text{out}} = \frac{1}{2} S_{0_{in}} = \frac{1}{2} I_{in}. \quad (1.40)$$

Moreover, by applying Equation 1.24 with the parameters obtained in Equation 1.39 one gets the angle of linear polarization ψ of the exiting beam equals to 0° , i.e. the exiting beam is horizontally polarized. Also, by applying Equation 1.22 in the same parameters, one verifies that the beam is fully polarized as the DOLP equals 1.

In reality, a nonideal linear polarizer is a diattenuator characterized by its maximum transmission: $T_{\text{max}} = q$, and its minimum transmission: $T_{\text{min}} = r$, with $r > 0$, i.e. the transmission coefficients of a polarizer are never perfect. In the general case, the Mueller matrix of a θ -oriented and nonideal polarizer is written as follows [12]:

$$\mathbf{M}_{\theta, q, r} = \frac{1}{2} \begin{bmatrix} q+r & (q-r)\cos(2\theta) & (q-r)\sin(2\theta) & 0 \\ (q-r)\cos(2\theta) & (q+r)\cos^2(2\theta) + 2\sqrt{qr}\sin^2(2\theta) & (q+r-2\sqrt{qr})\sin(2\theta)\cos(2\theta) & 0 \\ (q-r)\sin(2\theta) & (q+r-2\sqrt{qr})\sin(2\theta)\cos(2\theta) & (q+r)\sin^2(2\theta) + 2\sqrt{qr}\cos^2(2\theta) & 0 \\ 0 & 0 & 0 & 2\sqrt{qr} \end{bmatrix} \quad (1.41)$$

1.4.2 Retarder or Waveplate

A retarder consists of any polarization element designed to produce a specified phase difference between the exiting beams for two orthogonal incident polarization states. Retarders are constructed out of a birefringent material. Two particular cases of retarders are commonly met:

- The half-wave plate which introduces a phase shift of π rad in the propagation direction of linearly polarized light, equivalent to an optical path of half a wavelength ($\lambda/2$). It can be used as a polarization rotator.
- The quarter-wave plate, which introduces a phase shift of $\pi/2$ rad equivalent to an optical path of quarter wavelength ($\lambda/4$); making it possible to convert linearly polarized light into circularly polarized light and vice versa. The Mueller matrices of the said elements, for a θ -oriented fast axis, are respectively:

$$\mathbf{M}_{\lambda/2}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(4\theta) & \sin(4\theta) & 0 \\ 0 & \sin(4\theta) & -\cos(4\theta) & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \quad (1.42)$$

$$\mathbf{M}_{\lambda/4}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2(2\theta) & \sin(2\theta)\cos(2\theta) & -\sin(2\theta) \\ 0 & \sin(2\theta)\cos(2\theta) & \sin^2(2\theta) & \cos(2\theta) \\ 0 & \sin(2\theta) & -\cos(2\theta) & 0 \end{bmatrix}. \quad (1.43)$$

Contrary to polarizers, the spectral behavior of waveplates is highly wavelength dependent and has to be taken into account.

1.4.3 Polarization rotator

A polarization rotator is an optical device that modifies the angle of polarization ψ of a linearly polarized light beam without modifying its ellipticity of polarization ε . The rotation matrix along the axis $(0, 0, 1)$ in a 3-dimensional reference frame, rotating the AOP by an angle $\theta = \Delta\psi$ is as follows:

$$\mathbf{M}_{\text{rot}}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) & 0 \\ 0 & -\sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (1.44)$$

Polarization rotators of linearly polarized light can be found in applications of modern laser optics as laser beams tend to be linearly polarized, and as a result, it is often necessary to rotate the original polarization to its orthogonal state [13].

1.4.4 Depolarizer

A depolarizer is an optical device used to scramble the polarization of light, useful when unwanted polarization of the input of an optical system may cause errors in the system's output. An ideal depolarizer would output randomly polarized light whatever its input, but all practical depolarizers produce pseudo-random output polarization. A pure depolarizer can be represented by the matrix:

$$\mathbf{M}_{\text{dep}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{bmatrix}, \quad (1.45)$$

where $|a|$, $|b|$ and $|c| \leq 1$.

1.5 Polarimetry

Polarimetry is the sensing and interpretation of the polarization state of light coming from a target, i.e. the measurement of polarization properties of light beams (e.g. Stokes Vector) and samples (e.g. the Mueller matrix). The basic scientific instrument used to measure these various properties is referred to as a polarimeter. Many designs have been proposed for these optical instruments, being composed of a setup of polarization and sensing elements that depending on its purpose can be classified as:

- Polarization State Generator (PSG): consisting of a source of well-defined polarization states (linear, elliptical, circular or any other state, normally used as a referential measurement.
- Polarization State Analyzer (PSA): consisting of a specific set-up of polarization elements, optical elements (lenses, mirrors, etc.), and radiometric elements that carry out the sensing of a polarization component of an incoming beam. A PSA able to sense the full polarization information is characterized by a four-component analyzer vector $\mathbf{A}$, analogous to the Stokes vector, which in turn has the same corresponding values of the first row of the PSA's Mueller matrix as denoted in Equation 1.29, as follows:

$$\mathbf{A} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \end{bmatrix} = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 \end{bmatrix}. \quad (1.46)$$

PSGs and PSAs often use the same polarization modulation techniques.

1.5.1 Mueller Polarimeters

The sample-measuring Mueller polarimeters allow estimating the 16 components of the Mueller matrix of the observed element with the purpose of describing the polarizing behavior of a sample traversed or reflecting by a polarized light probe. Mueller's polarimetry

consists in illuminating the sample under test with four different polarization states (both linear and circular) generated by a PSG, and measure the output Stokes vectors with a PSA.

Let $\mathbf{B}$ be the 4x4 matrix whose rows are the Stokes vectors of the analyzing states, let $\mathbf{G}$ be the 4x4 matrix of which columns are the Stokes vectors output by the PSG and let $\mathbf{M}$, be the Mueller matrix of the target. Then, the measured intensity values can be synthesized in a matrix $\mathbf{I}$ that obeys the relation:

$$\mathbf{I} = \mathbf{B}\mathbf{M}\mathbf{G} . \quad (1.47)$$

If matrices $\mathbf{G}$ and $\mathbf{B}$ turn out invertible, the matrix $\mathbf{M}$ can be calculated by isolating it from Equation 1.47 such as:

$$\mathbf{M} = \mathbf{B}^{-1}\mathbf{I}\mathbf{G}^{-1} . \quad (1.48)$$

Regardless of the polarimeter technology used, a Mueller polarimeter will need to acquire several images sequentially by illuminating the scene with different states of polarization. It is therefore difficult to measure the Mueller matrix of a moving scene.

1.5.2 Stokes polarimeters

It is a type of light-measuring polarimeter intended to determine the Stokes vector parameters of a beam of light or some of its derived polarimetric characteristics, such as the degree of polarization, or the elliptical parameters. Stokes polarimeters can be divided in two categories:

- Linear-Stokes Polarimeter: when it contains only linear polarizers, and it is capable of determining the first three Stokes vector components.
- Full-Stokes Polarimeter: in which a retarder (other than a half-wave retarder) is added to capture both linear and circular polarization components.

A Stokes polarimeter can quantitatively estimate a Stokes vector by combining the measurement of the polarization of light according to the projection of different polarization state analyzers. The intensity measurement I taken by the PSA (characterized by the analyzer vector of Equation 1.46) relatively to a polarization state $\mathbf{S}$, is the dot product:

$$I = \mathbf{A}\mathbf{S} . \quad (1.49)$$

To carry out a Stokes vector measurement, the polarimeter must perform a series of measurements taken with a set of Q polarization state analyzers which can be multiplexed in time, frequency, or space. It is assumed that during the whole measurement process, the incident Stokes vector is the same for all PSAs. A Q-by-4 analyzer matrix² $\mathbf{W}$ can be defined by stacking each row containing an analysis vector $\mathbf{A}_Q$. By gathering the intensities

²also called measurement/synthesis/analysis/instrument/coefficient/reduction matrix.

measured into an intensity vector $\mathbf{I}$, we obtain a linear relationship between the vector $\mathbf{I}$ and the Stokes vector $\mathbf{S}$ as follows:

$$\mathbf{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_Q \end{bmatrix} = \mathbf{W}\mathbf{S} = \begin{bmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ \vdots & \vdots & \vdots & \vdots \\ a_{Q-1,0} & a_{Q-1,1} & a_{Q-1,2} & a_{Q-1,3} \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}. \quad (1.50)$$

The analyzer matrix $\mathbf{W}$ is characterized by the number of polarization state analyzers Q oriented at different angles θ_i , $i \in [1, Q]$. The analyzer matrix can become nonsingular from $Q = 4$. For $Q = 4$, by isolating $\mathbf{S}$ from Equation 1.50 one obtains the Stokes vector; by using the classical inverse operator as:

$$\mathbf{S} = \mathbf{W}^{-1}\mathbf{I}. \quad (1.51)$$

If $Q > 4$, the analyzer matrix $\mathbf{W}$ cannot be inverted, it is possible to use the Moore-Penrose pseudo-inverse estimator (+) in a way that Equation 1.51 can be written as:

$$\mathbf{S} = \mathbf{W}^+\mathbf{I}, \quad (1.52)$$

where $\mathbf{W}^+ = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T$ (the superscript T denotes matrix transposition). The Stokes estimation can be either calibrated or uncalibrated. In case it is uncalibrated, the estimation is made from an analyzer matrix $\mathbf{W}$ which is considered to be ideal, and will be noted $\mathbf{W}_{\text{ideal}}$. If $\mathbf{W}$ is not 4-ranked, i.e. the Q polarization analysis states do not form a basis in the 4D Stokes space, such an inversion process is not possible.

It has been shown that the Q polarization state analyzers oriented at different angles θ_i , with $i \in [1, Q]$, must be evenly distributed over the half-circle to optimize the polarimetric performance [14], such that:

$$\theta_i = \theta_0 + \frac{(i-1) \times 180^\circ}{Q}. \quad (1.53)$$

where θ_0 is an offset in respect to the x axis. If only linear polarization is considered, a subset of the first three Stokes components can be used and $Q = 3$ polarization states can be sufficient.

1.6 Conclusion

The theory of polarization of light and polarimetry, both Mueller and Stokes, can be applied to different areas in science and engineering such as: ellipsometry, spectropolarimetry for chemical applications, astronomy, radar, to name a few. The next chapter will be focused on polarimetry applied to imaging systems, dealing with the practical structures that allow a user to implement such systems and how they can be optimized. Finally, the chapter will be focused on imaging Stokes polarimeters, emphasizing the division of focal plane architecture, which is the main topic of this manuscript.

Chapter 2

Polarimetric Imaging

Polarimetric imaging is a special case of general polarimetry that seeks to obtain 2D images carrying information on the polarization of the light emitted or scattered by a scene, unlike point detection polarimetry. This technique makes it possible to emphasize spatially the polarimetric properties of objects that are useful to reveal information not visible with classic luminance or color imagery. Polarization analysis using imaging polarimetry instruments gets a growing interest in imaging applications such as classification of materials [15], 3D inspection and reconstruction [16], image dehazing [17], to name a few.

As discussed in the previous chapter, polarimetry can be classified as Stokes or Mueller. This chapter deals with both but is focused on the study of the Stokes imaging architecture, that is to say, light-measuring imaging systems that allow a user to get the spatial measurement of Stokes vector. Additionally, these devices can estimate the polarimetric parameters (degree and angle of polarization) of the reflected, scattered, or transmitted light. After a comparison of the different technologies, the chapter focuses on the division of focal plane architecture, because it is the main subject which will be discussed in the next chapters.

2.1 Mueller imaging architectures

This section describes the basics of the setup used to measure Mueller matrix elements. Since the Mueller matrix is a function of wavelength, angle of incidence, and location on the sample, new measures should be performed when these parameters change. In general, a sample-measuring Mueller imager consists of a source, a polarization state generator (PSG), the sample, a polarization state analyzer (PSA) pixelated detector.

The characterization of a Mueller imager consists in estimating the matrices: $\mathbf{W}$ (of which columns are the Stokes vectors output by the PSG) and $\mathbf{B}$ (of which rows are the Stokes vectors of the analyzing states), as precise as possible from measurements taken with referential samples of known polarizing properties, so that, the application of Equation 1.48 delivers a calibrated value. Until now, different methods have been studied to determine the measurement matrices in both transmission and reflection configuration [18, 19].

2.1.1 Mueller imaging in reflection configuration

Figure 2.1 shows the scheme of a general Mueller imager in reflection configuration. To perform calibration, reflection samples may be placed between the PSG and PSA. The linear polarizers may be placed between the PSG and the reflection sample, or between the PSA and the reflection sample. Under this scheme, different methods have been studied. Firstly, the Standard Eigenvalue Calibration Method (SECM) proposed in [20] is devoted to the complex and critical calibration of the PSG's, imaging polarimeters and Mueller matrix ellipsometers. The SECM assumes that the reference samples are ideal polarizer and reflective components. Therefore, to improve the calibration results, an Extended Eigenvalue Calibration Method (EECM) has been proposed in [18] to model the reference samples in a more general way. The SECM has been implemented to calibrate different Mueller polarimeters such as ones using a photoelastic modulator or a division of time prism and liquid variable retarders [21], as well as variants of the method to calibrate imaging polarimeters that use high numerical aperture optics [22], and also for the characterization of transparent biaxial samples [23].

In practice, intensity measurements are perturbed by noise. However, none of the above variants of the EECM takes into account the presence of noise in data. To this effect, the Maximum Likelihood (ML) calibration method proposed in [18] takes into account the statistical properties of noise in an optimal way, with a higher estimation accuracy and being suitable for any kind of noise source and not only applicable for the additive white and Gaussian noise.

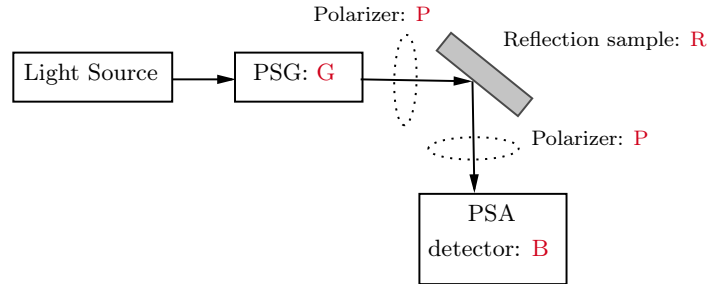


FIGURE 2.1: Scheme of the Mueller imager reflection configuration for calibration.

2.1.2 Mueller imaging in transmission configuration

Figure 2.2 shows the scheme of a general Mueller imager in transmission configuration. To perform calibration, reference samples (linear polarizer and retarder) are placed between the PSG and PSA to perform the intensity measurements. The transmission configuration can achieve the performance of the reflection configuration. The ML calibration method [18] is identical to that in reflection configuration. The calibration procedure has been simplified, so it is possible to perform the calibration without a retarder, and only with polarizers [19].

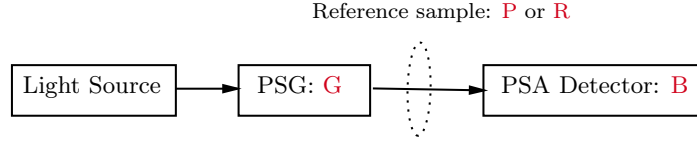


FIGURE 2.2: Scheme of the Mueller Imager in Transmission configuration for calibration

2.2 Stokes imager architectures

Stokes imagers systems can be categorized as either active or passive. If the polarimetric properties of the illumination source are known and controlled by a polarization state generator [24], the setup is considered to be active. On the contrary, a Stokes imager is said to be passive if there is no system to control the polarization of the illumination of the scene, therefore it is suitable for most natural light sources which are mainly depolarized (such as the sunlight), or they exhibit linear polarization after reflection. In the following, the manuscript will be focused on passive polarimetric imaging.

It has been shown that in the presence of additive Gaussian noise and for 4 intensity measurements, the conditioning of the analyzer matrix $\mathbf{W}$ is optimal when the analysis vectors form a regular tetrahedron on the Poincaré sphere [25, 26]. More intensities can be acquired to increase the signal-to-noise ratio, but at the cost of the acquisition time. In practice, these intensities are not necessarily I_0 , I_{45} , I_{90} , I_{135} , I_l , and I_r , as in Equation 1.15.

Polarimeters can be also categorized into scanning or snapshot devices, none of which is exempt of trade-offs and optical imperfections. Scanning polarimeters include the Division-of-Time polarimeter (DoT), whereas snapshot polarimeters include the Division of Amplitude (DoAmP), Division of Aperture (DoA) and Division of Focal Plane (DoFP) [27, 24]. These four main architectures will be explained as follows.

2.2.1 Division-of-Time imager (DoT)

This architecture relies on the principle of time-multiplexing sensing with the same polarization elements. It allows the acquisition of several polarization states by dividing the captures into different time slots [24].

Division-of-Time polarimeters can be categorized as rotating-element polarimeters [28]. For example, a single rotating polarizer allows identifying the first three components of the Stokes vector. Besides, Figure 2.3 depicts a rotating retarder, allowing the sensing of the circular components. In this configuration, the only rotation needed is that of the retarder. The mechanical DoT imager variant is relatively straightforward to implement. Among the potential disadvantages of this approach, there is the limitation when capturing a scene with movement (with either camera or object movements). Thus, these scanning devices produce image artefacts (often called "ghost artefacts") when fusing the sequential measurements into a polarimetric image. The DoT is therefore not suitable for measuring dynamic scenes.

The sequential commutation of the polarization projections to the sensor can also be carried out electrically by means of liquid crystal modulators as shown in Figure 2.4.

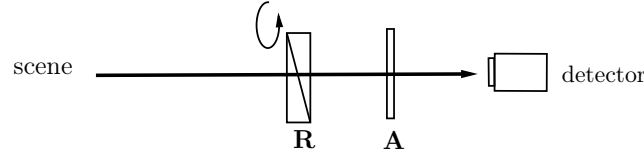


FIGURE 2.3: Schematic representation of a mechanical DoT imager using a rotating retarder plus fixed polarizer. A is a polarizer and R is a retarder

From a physicochemical point of view, a liquid crystal is a special type of state of matter that has the properties of the liquid and crystalline solid phases [29]. Depending on the type of liquid crystal, it is possible that the molecules have freedom of movement in a plane, but not between planes. The arrangement of molecules plays a large role in the polarimetric properties of liquid crystals, and therefore, they can be implemented as pure dephasers, polarization rotators or a combination of them. Liquid crystal cells make it possible to electrically modulate the polarization at frequencies which cannot be achieved with mechanically-oriented polarization elements. This method is thus much more adapted to capture dynamic scenes with a high framerate.

Different methods to calibrate image Liquid-Crystal Stokes polarimeters have been developed [30], under no assumption on the optical components, and therefore, reducing significantly the fabrication requirements. In a work carried out at the IRIMAS, Gendre *et al.* [6, 31] showed that a suitably controlled ferroelectric liquid crystal cell can be calibrated to capture the entire polarization state at approximately 200 frames per second but with a rather poor conditioning.

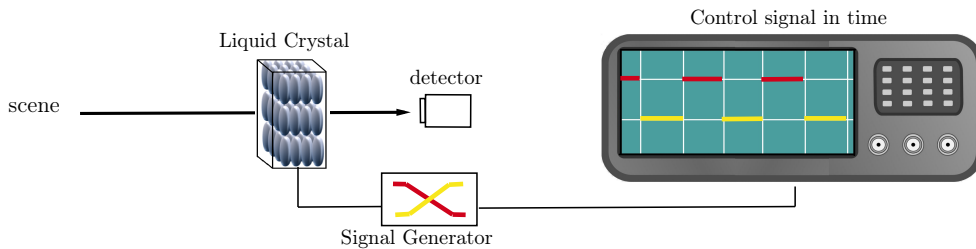


FIGURE 2.4: Schematic representation of a liquid-crystal DoT imager. The polarization projections to the sensor are commuted electrically.

2.2.2 Division-of-Amplitude imager (DoAm)

In division of amplitude polarimetry, the incident beam is split into several beams with different polarization states, before to be analyzed by various PSA [32]. This method has the advantage of being able to perform all the measurements simultaneously [28]. On the downside, one needs as many detectors as the number of Stokes components that are intended to be estimated. In addition, it is essential to synchronize the cameras, as well as to perform alignment either mechanically or by processing the images once the acquisition has been made. For example, a schematic diagram of a four channel DoAm [33] is shown in Figure 2.5.

To perform calibration on the DoAm architecture, a method that enables to calibrate the geometric and linear polarization properties of the imager has been developed [34], with a simple tablet that is manually rotated in front of the camera. The full Stokes vector is estimated simultaneously by combining the measurement of the four detectors. Size and investment in hardware are to be considered. The DoAm architecture has been used in the magnetic field research in the so-called Advanced Stokes Polarimeter (ASP) [35]. As a result, several methods have been developed to perform full Stokes calibration in the ASP.

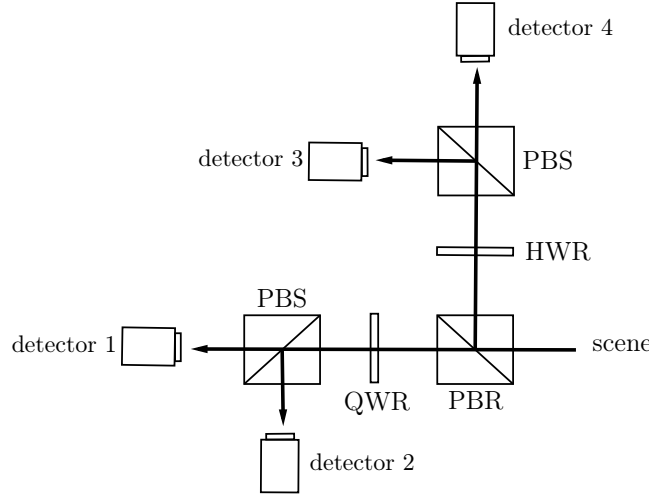


FIGURE 2.5: Full-Stokes DoAm imager [33]. PBS is a polarization beam splitter, allowing each detector to sense distinct polarization states. PBR is a polarization beam rotator, QWR is a quarter-wave retarder, and HWR is a half-wave retarder.

2.2.3 Division-of-Aperture imager (DoA)

The DoA architecture uses a single photosensitive imager, and it is based on an array lens set that produces four identical images on a single focal plane array from a unique aperture. The collimation optic forms an image of the objective lens aperture of the microlens array in a way that the incident light on the objective lens is evenly divided across the four cells. A micropolarizer is placed after each microlens to measure four different polarization linear states across the object, namely, 0, 45, 90, and 135 degrees (polarizer array) [36]. The primary drawback of this method is the loss of spatial resolution and the volume and weight of the additional optics [24]. Figure 2.6 depicts a schematic diagram of a division-of-aperture polarimeter [36].

2.2.4 Division-of-Focal-Plane imager (DoFP)

The DoFP architecture relies on the principle of shared-sensor and different micropolarization elements oriented at Q angles. It allows the user to acquire several polarization states by dividing the capture into different focal plane slots. The primary drawback of this method is the loss of spatial resolution. However, it is suitable for dynamic scenes as all the polarization information is captured in a snapshot way. This kind of architecture has

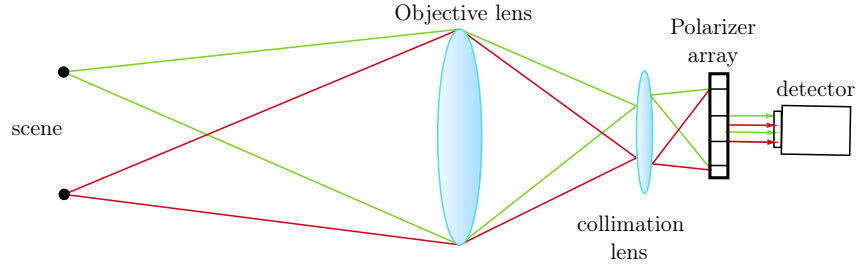


FIGURE 2.6: Concept design for Division of Aperture polarimeter [36].

also been considered in spectral imaging [37, 38]. Figure 2.7 depicts a schematic diagram of a Division of Focal Plane polarimetric imager.

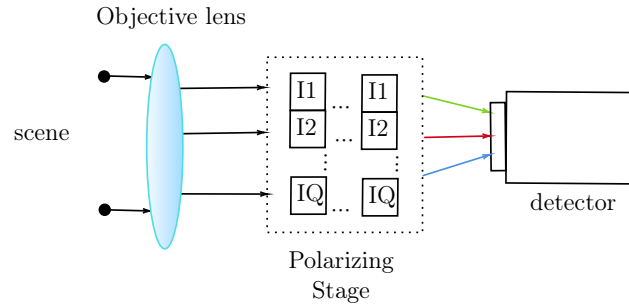


FIGURE 2.7: Design concept of the division of focal plane imager. Q measurements taken by means of one micropolarizer place on each pixel is repeated to cover all the photodetector array.

2.2.5 Comparison of Imaging Polarimetry architectures

Table 2.1 gathers the characteristics of the polarimeters that we have reviewed earlier.

Technology	Challenges	Benefits
DoT imager [28]	-Moving parts (mechanical variant). -Scene must be static. - Limited switching speed.	-Simplest design. -Inexpensive (mechanical variant).
DoAm imager [32]	-Expensive. -Maintenance effort. -Many variables to control.	-Simultaneous capture. -Full resolution capture.
DoA imager [36]	-Loss of spatial resolution -Volume and complex optics.	-Simultaneous capture. -Suitable for Full Stokes sensing.
DoFP imager [24]	-Loss of spatial resolution. -Fabrication and alignment difficulties.	-Simultaneous capture. -Reduced size. -Currently evolving.

TABLE 2.1: Comparison of Imaging Polarimetry Architectures.

For many applications, the DoFP imager can be considered as a very interesting choice due to its increasing accessibility and its continuous improvements. Another remarkable advantage of the DoFP imager is that it allows the user to measure polarization in a single acquisition, so it is suitable for video acquisition, and moving scenes. Moreover, these devices lead to a simpler design and higher compactness than other snapshot devices. The

DoFP imager is the architecture that we will detail extensively in section 2.3, as it is the focus of this work.

2.2.6 Influence of noise on estimation precision and analyzer matrix optimization of Stokes imagers

There exist several approaches to model the noise in both the spectral and the polarimetric imaging, some approaches consider noise-free images, which is not a realistic assumption. The estimation precision due to noise allows guiding decision in design of imaging polarimeters as well as the quantification of the limits when measuring the Stokes vector and polarimetric parameters.

Imaging noise model

In this manuscript, the term noise refers to any temporal and random variation of the luminance at each pixel. Any stationary spatial non-uniformity between different pixels to a uniform or zero irradiation, namely, the so-called "Fixed Pattern Noise", as well as defective pixels, particularly, partial pixel defects, is out of the scope of this section, as these are counteracted by calibration in Chapter 3.

Temporal noise can be described in two different domains: spatially and temporally. In the temporal domain, the values in the same pixel can be different from one image to another. Notably, when recording video of a static scene; it can be seen as a dynamic grainy effect. Also, in the spatial domain it can be perceived in a region of uniform light properties (either spectral or polarimetric), the noise will result in fluctuations of the digital values among different pixels; giving the appearance of grainy artifacts. Temporal noise can be suppressed by averaging a high number of captures taken in sequential slots of time in a pixel-to-pixel fashion (typically between 100 and 400 images [39]), however, this procedure is only suitable for static scenes, having constraints analogous to the Division-of-Time imager architecture, as the image is prone to exhibit motion blur.

Figure 2.8 shows a block diagram of a noise model considering three sources of noise [39]. This model is positioned before any image processing correction stage. On the one hand, the Additive White Gaussian Noise (AWGN)¹ encapsulates several natural noise sources, such as the thermal vibrations of atoms in conductors that result in the temporal variation of the dark offset; being normal-distributed in time (Gaussian process) and having a uniform distribution of the spectral frequencies (white) that made up the additive signal. Mathematically, the AWGN is quantified by the (temporal) standard deviation of the signal measured with zero irradiation, and it is denoted by σ [in units of electrons].

On the other hand, the real signal has an intrinsic random variation that stems from the discrete nature of photons hitting the sensor. The varying number of photons determines the number of electrons generated (related to the quantum efficiency) and ultimately affecting the output digital number (related to the system's gain). This random variation

¹also called dark/ readout/ background/ read noise.

is known as Poisson Noise². Mathematically, the Poisson noise is quantified by the (temporal) standard deviation of the signal's magnitude (number of photons, number of electrons, or digital value). For a sufficient level of light, it has been shown that the Poisson Noise dominates over other noise sources, in both classical color [40] and monochrome polarimetric cameras [25, 41]. The final analog-to-digital conversion adds another noise source known as quantization noise, which is uniformly-distributed between quantization intervals and has a variance that has an inverse-square relationship with the digital resolution [42]. Table 2.2 summarizes the properties of the three sources of noise that have been discussed.

Noise	Source	Distribution
AWGN	Internal natural phenomena	Gaussian/Normal
Poisson	Discrete nature of photons	Poisson
Quantization	Analogue to digital conversion	Uniform

TABLE 2.2: Comparison of the properties of the additive Gaussian, Poisson, and Quantization noise.

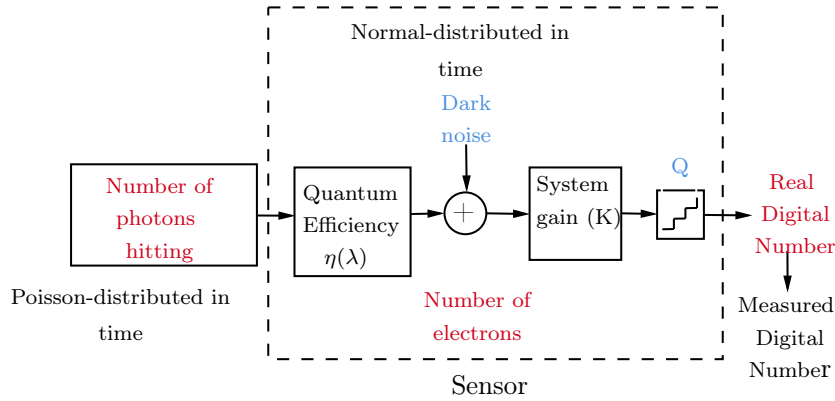


FIGURE 2.8: Block diagram of the noise model considered for the PFA imager. The signal-independent dark noise is added to the signal with a value that is normal-distributed, whereas the signal-dependent shot noise scales the magnitudes (in red) by a factor related to a Poisson process. The quantization noise Q is added during the ADC conversion.

Impact of the mathematical operators in the presence of noise

When determining the Stokes parameters, the additive noise is propagated in the inversion procedure of Equations 1.51 or 1.52 and thus multiplied by the pseudo-inversed analyzer matrix $\mathbf{W}^+$. As a result, it is important to determine the optimal design of polarimeter structure, i.e. the number of polarization state analyzers Q and the offset among the chosen measurement angles which ensure the best precision and the smallest noise [9]. Nevertheless, any criterion has a trade-off with the feasibility of cost, complexity and even power consumption, that may lead to preferring less optimal structure designs [43] as well

²also called Poisson-shot/ shot/ photon/ multiplicative/ scale noise.

as guiding the decision to choose a measurement system adapted to the parameter of interest.

It has been established that in the presence of additive noise, the pseudo-inverse estimator is the maximum likelihood estimator of the Stokes vector [25]. Besides, in the presence of Poisson shot noise (PSN), which is unavoidable as it is associated to the particle nature of light, the pseudo-inverse estimator is a simple decision rule that can be approximated as unbiased [9]. Ultimately, the said estimator may be not optimal for other sources of noise but can be chosen for the sake of simplicity.

Moreover, it is assumed statistical independence between the additive and Poisson noise as well as among the measured values in different pixels at the same time, and therefore any parameter calculated out of the said values, such as the Stokes components.

Metrics of estimation precision in polarimetric imagers

Stokes vector optimization based on the analyzer matrix performance has been evaluated with metrics such as the condition number (CN) [44, 45], the Equally Weighted Variance (EWV) [9, 46], the signal-to-noise ratio [47], the matrix determinant [48], and a priori content information [49].

The equivalence of the EWV, CN and the determinant of the associated Gram Matrix has been studied [50], proving that the characterization of the said metrics are optimal when the set of measurements are spatially distributed on the Poincaré sphere when they describe a so-called "spherical 2-design". A spherical t -design is a set of N points on the surface of the n -dimensional sphere (in this context, $\mathbb{R}^3$ and N referring to the Q number of intensity measurements), for which the normalized integral of any polynomial of degree t or less is equal to the average taken over the N points [51]. Regular and convex polyhedra known as Platonic solids (e.g, tetrahedron, octahedron, cube, icosahedron, dodecahedron) fall into this category as well other nonconvex designs.

The equivalence of the different optimization metrics has been reexamined, arguing that the EWV has an advantage over the others as it bears an actual statistical meaning, as it sums up the variances among Stokes components. Mathematically, the *EWV*, is expressed as [9]:

$$EWV = \text{trace}[\Gamma^{\mathbf{S}}] = \text{VAR}[S_0] + \text{VAR}[S_1] + \text{VAR}[S_2] , \quad (2.1)$$

where Γ denotes the autocovariance matrix operator and $\text{VAR}[\cdot]$ denotes the variance. Although the noise is mathematically quantified by the standard deviation, it is usually expressed in terms of the variance as variances sum up. Therefore, the *EWV* satisfies the property of superposition [52] to any source of noise. As a result, the total value of the *EWV* is a variance itself, and it can be in turn combined between different noise sources such dark and Poisson, by assuming they are uncorrelated. Therefore, the total *EWV* can be written as:

$$EWV_{total} = EWV^{add} + EWV^{poi} = \text{trace}[\Gamma^{add}] + \text{trace}[\Gamma^{poi}] , \quad (2.2)$$

where EWV_{add} is the equally weighted variance in the presence of only additive noise and EWV_{poi} is the equally weighted variance in the presence of only Poisson noise. This additive property has been used to determine the estimation precision of general Stokes polarimeters [53] as well as the specific case of monochrome division-of-focal-plane imagers in the presence of both Poisson shot and additive white Gaussian noise [54], including the impact of the polarization nonidealities. Moreover, in the same microgrid architecture, the polarimetric parameters' estimation precision has been improved to include the influence of spatial nonuniformities, resulting in a more accurate estimation of the polarimetric parameters [55].

The estimated Stokes vector is expressed in terms of the number of photoelectrons, which in turn is proportional to the number of photons that impinge the sensor. In general, it is assumed that each of the Q measurements has the same exposure time and the total amount of photons that hit the sensor is fixed [9, 56]. Based on this, different analytical approximated close-form expressions have been derived to determine the estimation variance of the Stokes vector and the polarimetric parameters: DOP , AOP and EOP [57, 43]. These models make it possible to obtain the fundamental limits for the estimation errors, as a function of both the number of measurements, noise level and external conditions, such as intensity and polarization states of light. Ultimately, if any of the close-form expressions of the variance are not valid, e.g., due to incident light not sufficiently polarized, the particular probability density functions must be resolved and one must obtain the estimation parameters out of them.

For the polarimetric analysis considered in this manuscript, is excluded from the model, any erratic noise source, e.g., due to thunderstorms as well as the quantization noise.

2.3 Polarization Filter Array imager technology

The DoFP Stokes imager relies on the Polarizer Filter Array (PFA) technology, which is a special case of the general filter array imaging principle [58, 59].

The PFA was first patented in 1995 [60], but most of the practical implementations and technology advances have been made since 2009 following various manufacturing processes [61]. The PFA, also known as microgrid polarimetric array, is inspired from the Bayer scheme used in color imaging. The PFA sensor's polarizing stage consists of a matrix of micro-polarizing elements spatially arranged in a lattice as seen in Figure 2.9. The polarizing matrix is obtained through the addition of different material structure such as: aluminum nanowire [62, 63], and liquid crystal polymer [64, 65].

The most famous spatial arrangement of polarizers, which is practically implemented in commercial devices, is the Chun pattern [66] (see Figure 2.9). In this case, the primitive cell, called the superpixel, consists of four linear polarizers with relative orientations evenly distributed in $[0^\circ, 180^\circ]$, that is 0° , 45° , 90° , and 135° , which satisfies the optimization criterion in [14] pointed out in Equation 1.53, with $\theta_0 = 0^\circ$ (when the 0° -oriented channel lined up with the x axis). Each polarization direction is located diagonally to its orthogonal counterpart as suggested by Chun [66]. This standard pattern was introduced in 1994, but

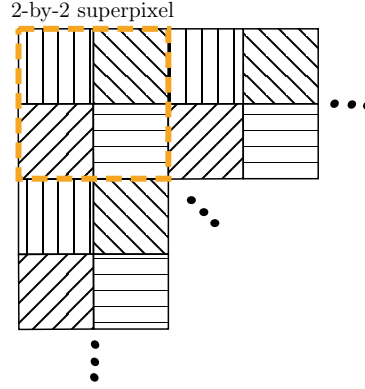


FIGURE 2.9: Schematic representation of a Polarizer Filter Array (PFA) or micropolarizer grid. Here, a 2-by-2 super-pixel is made up of 4 pixels with micro-polarizers oriented at 0° , 45° , 90° and 135° . Every quadrant is located diagonally to its orthogonal counterpart. This refers to the Chun pattern [66].

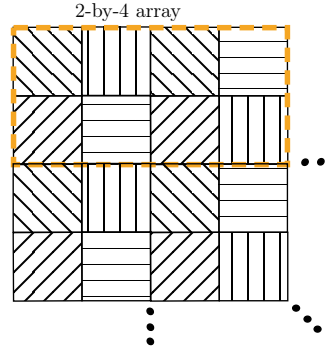


FIGURE 2.10: Schematic representation of an improved Polarizer Filter Array (PFA) or micropolarizer grid. Here, a 2-by-4 array is made up of 8 pixels with micro-polarizers oriented at 0° , 45° , 90° and 135° grouped into two subpatterns. Every quadrant is located laterally to its orthogonal counterpart. This refers to the LeMaster pattern [67].

extended arrangements have been proposed to provide a better spatial resolution [67] (see Figure 2.10).

The PFA is aligned in a one-to-one fashion to the imager's focal-plane array (see Figure 2.11), so that each pixel analyzes a particular direction of polarization. The acquisition of intensities through the four linear polarizers makes it possible the estimation of the first three Stokes vector components of the input light; i.e. the Chun PFA imager is an intrinsic linear-Stokes polarimeter. The polarimetric calculation unit of the DoFP imager is the 2-by-2 super pixel (also called a polarimetric pixel).

2.3.1 Full Stokes vector with a PFA imager

Practical implementation of full-Stokes PFA instruments is at its very beginning. Intrinsically, the Chun PFA imagers are not sensitive to circular polarization, so they only allow a user to measure the linear part of the Stokes vector. To measure the full Stokes vector in a snapshot way, at least one of the pixels of the superpixel must be sensitive to circular

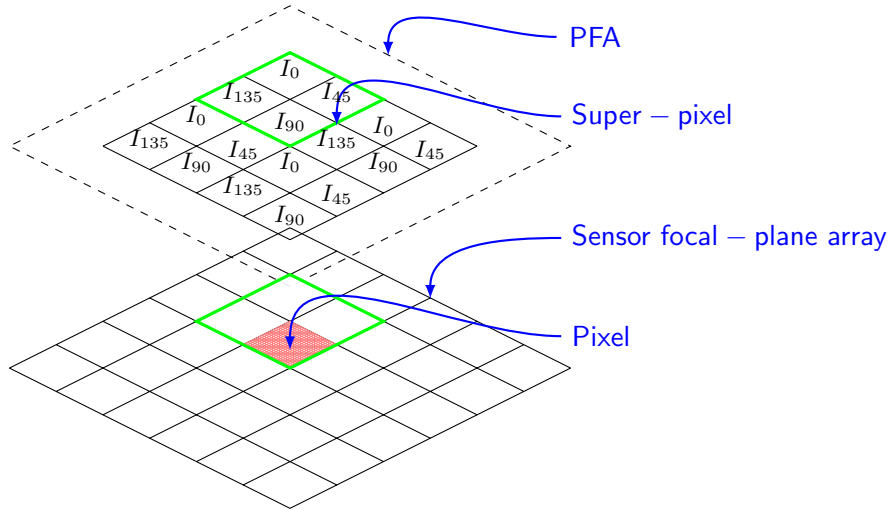


FIGURE 2.11: Cell-to-cell coupling of a Polarizer Filter Array (PFA) to the imaging system's focal-plane array (FPA). Each optical element corresponds to a pixel of the imager, so that each pixel analyzes a particular direction of polarization. The assembly presented in this figure only allows the analysis of linear polarization.

polarization. Some existing specific patterns of micro-polarizers make it possible to measure the complete Stokes vector [68]. These structures have been designed by gathering 2-by-2 superpixels with 3 linear polarizers oriented at 0, 45, and 90°, along with a circular polarization filter which is a microretarder, and consequently, reducing the operation at a single wavelength as this polarizing element is not broadband. Likewise, other PFA full-Stokes sensor designs with larger super pixels have been proposed [69], as well as the use of liquid crystal polymer instead of wire-grid polarizer technology [64]. It requires fewer lithographic steps than the wire grid polarizer, and also removes the need for a uniform coating and fabrication of an additional microretarder layer [65].

An alternative way to measure the full Stokes vector with a PFA camera is the use of additional rotated birefringent optical elements in front of the PFA camera. Under this scheme (depicted in Figure 2.3), the imaging system loses the snapshot capability, carrying the constraints of DoT imagers. Examples of implementations are in [70, 71]. Nevertheless, an optimal configuration allowing the user to measure a full Stokes vector under this hybrid approach has been studied [54]. The study of circular polarization has several applications in active imaging, but in natural scenes, the circular component of light is commonly neglected. In the following, only linear PFA will be considered.

2.3.2 Assembly process variants of PFA sensors

When the micro-polarizer is made up of aluminum wires, the width and spacing between lines are of the order of ten to one hundred nanometers. These aluminum wires can be made by optical or electronic lithography [72]. Such wire-grid polarizers carry out the polarization by refraction, as well as typical macroscopic polarizers.

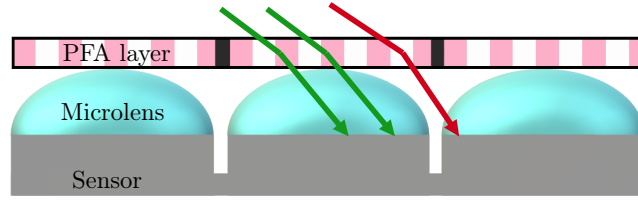
Optical crosstalk is an unwanted effect that occurs when light passes through a given micropolarizer at nonzero angle of incidence, but instead of only hitting the sensor pixel located directly behind the micro-polarizer, it also hits one or several contiguous pixels. From the PSA's standpoint, optical crosstalk is a significant source of polarimetric performance degradation, having a greater effect on the edges of the sensor where the light arrives at a greater angle of incidence [62], this phenomenon will be detailed in Chapter 4. As a result, the assembly process of the PFA and the photodetector pixel array has evolved to counteract this effect, so that two manufacturing variants are distinguished [73].

Figure 2.12 depicts the PFA to photodetector array assembly process. Subfigure 2.12(a) shows the earlier post-process assembly structure (on-glass) where some rays hit a photosensor with light filtered by the wrong micropolarizer and Subfigure 2.12(b) depicts the later in-process manufacturing structure (on-chip) in which the PFA is positioned just above the pixel array and below the microlens array, so that rays with high angles of incidence are refracted and hit the right photosensor, reducing the likelihood of crosstalk between neighboring pixels and improving alignment among the layers [74, 75]. The maturity of the latter manufacturing process has also an impact on the affordability of DoFP cameras, which has greatly increased, making them a competitive technology as compared to the other Stokes imager architectures. For color imagers, the market tends to reduce even more the size of the pixels, to get a higher spatial resolution. In polarimetry, one can wonder whether the reduction in pixel size can follow the same trend. It is known that the optical crosstalk has a more significant impact in the polarization domain than in the spectral domain. Additionally, both electronic and optical crosstalks depend on the pixel pitch [76, 77]. Thus, from these statements, one has to find a tradeoff between the quantum efficiency of pixels and the polarimetric measurement degradation when designing a high resolution polarimeter.

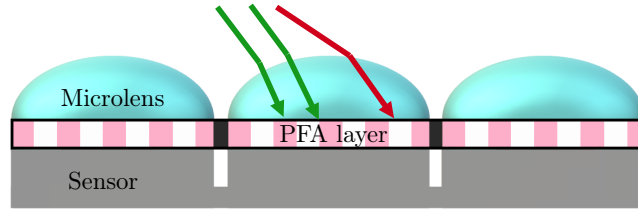
2.3.3 PFA Linear Imaging model

The Chun superpixel's analyzer matrix $\mathbf{W}$ can be synthesized by stacking each row containing an analysis vector $\mathbf{A}_i = \begin{bmatrix} a_0 & a_1 & a_2 & 0 \end{bmatrix}$ with $i \in [1, 4]$. We could then assume a unique spatial position for the four adjacent polarizers, i.e. a superpixel. To index each analyzer matrix, we use the notation $\mathbf{W}_j$, where j indexes the spatial position of the super-pixel. The intensity vector that gathers the measured signals by the polarimeter is $\mathbf{I}_j = \begin{bmatrix} I_0 & I_{45} & I_{90} & I_{135} \end{bmatrix}^t$. By substituting the said notation in Equation 1.50, one gets:

$$\mathbf{I}_j = \begin{bmatrix} I_0 \\ I_{45} \\ I_{90} \\ I_{135} \end{bmatrix} = \mathbf{W}_j \mathbf{S}_{in} = \begin{bmatrix} a_{0,0} & a_{1,0} & a_{2,0} & 0 \\ a_{0,45} & a_{1,45} & a_{2,45} & 0 \\ a_{0,90} & a_{1,90} & a_{2,90} & 0 \\ a_{0,135} & a_{1,135} & a_{2,135} & 0 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ 0 \end{bmatrix}. \quad (2.3)$$



(a) Post process assembly structure (on-glass). The red arrow represent a ray with a high angle of incidence that is filtered and hits an adjacent pixel.



(b) In process structure (on-chip). The microlens layer is placed above the PFA so that rays with high angle of incidence are refracted and hit the corresponding photosensor.

FIGURE 2.12: PFA to photodetector array assembly process variants.

Where the ideal linear analyzer matrix of a Chun superpixel defined as in Equation 2.3 equals to:

$$\mathbf{W}_{\text{ideal}} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}. \quad (2.4)$$

If the ideal case is assumed, the Equation 1.51 is written as:

$$\mathbf{S} = \mathbf{W}_{\text{ideal}}^+ \mathbf{I}_{\mathbf{j}}. \quad (2.5)$$

By combining the values of Equation 2.4 in Equation 2.5, one may obtain the Stokes parameters out of the intensity vector components as follows:

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} I_0 + I_{90} \\ I_0 - I_{90} \\ I_{45} - I_{135} \end{bmatrix}. \quad (2.6)$$

It can be seen that Equation 2.6 is a particular case of Equation 1.15 (linear polarization sensing). In a PFA imager, the Stokes estimation can be done with or without spatial interpolation algorithms, i.e. demosaicing algorithm. Thus, the Stokes vector can be estimated either by superpixel (without demosaicing), or by pixel (with demosaicing).

By representing the polarization spatial distribution of the four linear measurements obtained with a Chun pattern in the Poincaré sphere, one gets a regular polygonal design, namely a square as shown in Figure 2.13. It has been shown that regular polygonal designs on the equator of the Poincaré sphere minimize the variance of the estimation of angle of polarization [78]. As a result, the intrinsic design of the PFA imager is optimal to estimate this parameter. Moreover, it has been shown that such a regular polygonal design minimizes EWV or CN and therefore is the best configuration for linear polarization estimation [78].

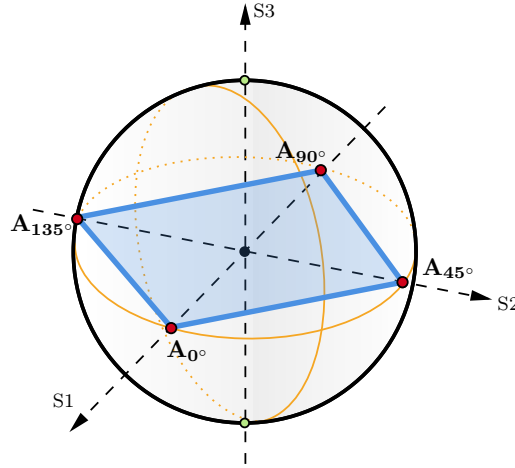


FIGURE 2.13: Geometrical depiction of the polarization states sensed by a 2-by-2 superpixel in a Chun-patterned PFA sensor. The point denoted by $\mathbf{A}_\theta$ corresponds to polarization state sensed in the θ -oriented PSA. The distribution turns out a regular polygon (square) whose vertices are on the equator of the Poincaré sphere.

2.3.4 PFA Analyzer Matrix Characterization

For DoT imagers, the analyzer matrix is often assumed to be the same for all pixels in the sensor. Contrary, for DoFP imagers, the analyzer matrix for each superpixel is different due to the limitations in nanomanufacturing process. In most cases, these matrices are not given by PFA manufacturers and should be estimated during a characterization step. The polarization characteristics of the camera can only be calculated indirectly by generating input light with known polarization states, and combining the camera intensity measurements.

Apart from the intrinsic sources of noise that can corrupt the camera signals, the estimation precision of the Stokes vector also depends on the precision and consistency of the analyzer matrix $\mathbf{W}$. Since a PFA sensor does not contain ideal polarization elements, the measurements are affected by component defects, misalignment of microelements, and deviations of the nominal micropolarizer's orientation axes, therefore, the PFA imager's measurements must be corrected. For this purpose, the matrix must be estimated to calibrate out these sources of inaccuracy. Nevertheless, differences between the measured $\mathbf{W}_{\text{est}}$ and the real analyzer matrix $\mathbf{W}_{\text{real}}$ exist, and will always be present, which may lead

to miscalibration [79]. Thus, the estimated Stokes vector from a miscalibrated polarimeter cannot be obtained with accuracy.

Several experimental setups have been proposed in the literature to perform the analyzer matrix characterization of PFA cameras. They are based on the use of a referential Polarization State Generator (PSG) in front of the PFA imager (acting as a Polarization State Analyzer). The PSG has been implemented under different configurations, either including only a single polarizer with an arbitrary light source [80], an integrating sphere and a rotating polarizer [81, 82, 83], or a nonuniform light source and a polarizer [73, 84]. Moreover, the relationship between the sensor orientation regarding the incident light [85], as well as impact of the polarization measurement during the characterization, such as the f-number and focal length influence [64, 86], have been considered. The number of parameters may also include photometric variables such as exposure time, conversion gain, and gamma correction [87].

In Chapter 3, the model presented in subsection 2.3.3 will be studied considering the non-ideality of polarizers with the purpose of using the characterized analyzer matrix to perform the calibration of PFA cameras.

2.3.5 Color-Polarization Filter Array imager

The Color-Polarization Filter Array (CPFA) is the color version of the monochrome PFA; which is capable of filtering linear polarization information in several spectral bands [88, 89, 90]. Both polarimetric and color images can be produced so that one can exploit both types of information jointly, allowing a wide range of applications. Some pre-processing [91] or data analysis [92] dealing with both spectral and polarization information have already been investigated in the literature.

In a CPFA camera, the output of each pixel intensity $I_{\theta,c}$, results from the sampling of the light intensity through one polarizing direction θ and one spectral band c . A common CPFA imaging device is a 12-channel sensor, which combines four polarization angles of analysis and three color channels, where $c \in \{r, g, b\}$. The polarizer angles are spatially arranged under the Chun pattern, whereas the three color filters are arranged under the Quad Bayer [93] spatial configuration (hereafter Quad Bayer/Chun). Figure 2.14 shows the schematic representation of the Quad-Bayer Chun of a CPFA sensor. Other spatial arrangements of vertically stacked color photodetectors approach (Foveon technology), have been proposed [94], mimicking the mantis shrimp visual system. Polarimetric models have also been developed for the calibration of spectral and polarization imagers [95].

The inter-channel and spatial correlations among spectral and polarization channels have also been investigated in [92]. As it was the case for color and spectral imaging [96], these data correlation results can help the definition of optimal spatial arrangement in the future, and lead to better spatial reconstruction, i.e. to preserve full-resolution images.

Conversely, Tu *et al.* [88] customized a color polarization camera under a unit cell consisting of a 2×2 Chun pattern, each of which is aligned to a 2×2 color filter (either red, green, or blue). This results in a 4-by-4 super pixel, hereafter Tu Bayer/Chun, shown in Figure 3.7. The calibration procedure is done by modeling the pixel's response behavior

such that every pixel of the camera has a polarization response for every color channel and the missing measurement of color reconstruction were completed using demosaicing algorithms. Tian *et al.* [97] implemented the Tu Bayer/Chun arrangement to propose a microscope application wherein the calibration function is specified such that the gain is the product of the characterized analysis vectors multiplied by the ideal values as demonstrated in [81] for a 2-by-2 superpixel. Vaughn *et al.* [98] introduced a spatio-temporal analysis to compare the Tu Bayer/Chun against the Quad Bayer/Chun concluding that the Tu Bayer/Chun has a better performance respect to the signal-to-noise ratio.

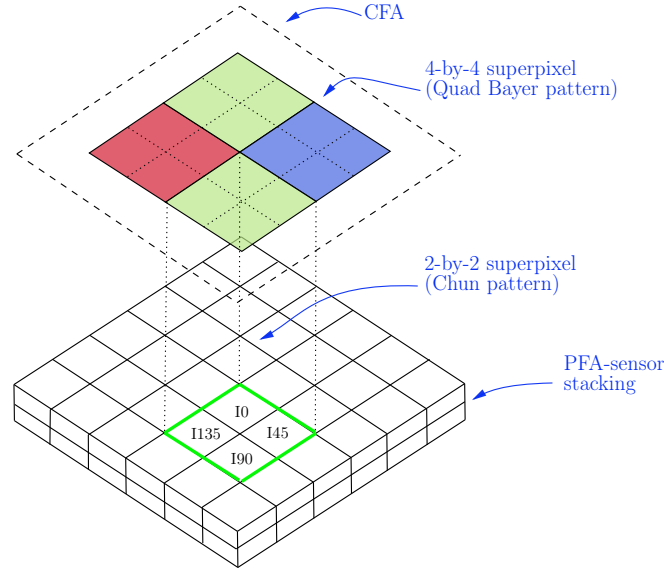


FIGURE 2.14: The CPFA sensor results from the coupling of a PFA-FPA stacking to a Quad-bayer Color Filter Array (CFA). Each 2-by-2 superpixel corresponds to one RGB color. The assembly presented in this figure only allows the analysis of linear polarization.

2.3.6 DoFP Commercial devices

There are different ways to manufacture a functional DoFP. Some companies, specialized in polarizing optics, are only developing the PFA layer, e.g. Moxtek Inc. [99], which manufactures the pixelated polarizers using nanowire-grid technology. The PFA are then post-process stacked to a standard imager to make the DoFP imager (on-glass).

In a higher level of implementation, some camera manufacturing companies sell the combination of the PFA with a focal plane sensor. One such company is Sony Semiconductor Solutions [100], which has greatly opened access to the market since 2018 with its IMX250 MZR/MYR-series on-chip sensors that were released for third parties camera manufacturers. From the user's perspective, other companies offer ready-to-use hardware and software DoFP systems. Here is a non-exhaustive overview of a few camera assemblers; which offer off-the-shelf Stokes imagers commercially available. It should be noted that certain information is sometimes difficult to access due to the confidential nature of defense applications in military agencies and, moreover, several models are customized on request.

- 4D Technology Corporation

The 4D Technology Corporation currently offers a DoFP micropolarizer camera featuring an on-chip 5 mega-pixel CMOS Sony IMX250 CMOS (Mono) Sensor [101]. The video rate is up to 75 frames per second (fps), and ensures a fast capture of a quickly changing scene. Optionally, they also offer a licensed software which provides a real-time display and the calculation of polarization parameters, including Degree of Linear Polarization, Angle of Linear Polarization, along with the linear Stokes components.

- LUCID Vision Labs

LUCID Vision LabsTM, Inc. offers a variety of DoFP micropolarizer cameras featuring on-chip sensors [102]. On the one side, they provide micropolarizer cameras featuring 5 mega-pixel Sony CMOS IMX250 MZR (with monochrome polarization filter array) and IMX250 MYR (with color polarization filter array). The frame rate is about 24 fps. More recently, this corporation has released the polarized CMOS sensors IMX264MZR/MYR (mono PFA/CPFA) yielding about the same performance as the IMX250MZR/MYR, but at a reduced cost. The Arena Software Development Kit (SDK) provides the access to a C++ library to use these cameras. The SDK supports the GigE Vision standard on either Windows, Linux, and ARM platforms.

- Noxant

This company build DoFP imager named NoxCam-Pola [103] featuring a polarimetric longwave infrared cooled sensor, and delivering a single image, containing polarimetric information in four directions together along with a infrared image. The spectral range is from 7.7 to 9.1 μm with a frame rate of up to 60Hz.

- Polaris Sensor Technologies, Inc.

This society offers a DoFP imager called Polaris Pyxis [104] featuring an uncooled VOx Microbolometer Array detector and working in the Long Wavelength Infrared that measures both the thermal and polarization content of an image in the waveband 7.5 to 13.5 μm with a frame rate of up to 30Hz.

- Teledyne FLIR

This company offers two DoFP imagers named Blackfly S. These micropolarizer cameras feature the 5 mega-pixel Sony CMOS IMX250 MZR (monochrome PFA) or IMX250 MYR (color PFA). The Blackfly S is available in GigE, USB3, cased, and board-level versions.

2.4 Conclusion

In this chapter, the study of polarization of light has been confined to polarimetric imaging of both Mueller and Stokes. In the first section, the different architectures have been

introduced, along with the state of the art of estimation precision, which have been developed mostly following a general approach. Then a comparison among the architectures is presented to justify why the division of focal plane imaging is the area of study of this manuscript, but it is clear that any choice is always a trade-off.

The second section is dedicated to the particular case of the polarization filter array imager, including detailed schematic representations, the different assembly processes and the mathematical linear model. It has also been seen that the PFA imager can be combined with other architectures such a division of time to measure circular polarization or with other imaging technologies such as the Bayer filter array to be able to perform spectral polarimetric imaging. It is worthy to mention that the PFA architecture is linked to a number of analyzer matrices that is proportional to the number of pixels in the captured image. The manufacturing and assembly process makes those matrices to be different spatially, and therefore they must be characterized individually. This characterization includes all the manufacturing imperfections or the nonidealities that lead to incorrect measurements and thus a polarimetric inaccuracy.

In the next chapter, the concept of calibration, i.e. the numerical correction of measurement, will be broken down. Moreover, the establishment of a consensus to the calibration setup that allows to perform the characterization process optimally will be investigated.

Chapter 3

Calibration and characterization of polarizer filter array Stokes imagers

3.1 Introduction

The end of Chapter 2 dealt with the sources of inaccuracies that affect the measurements in PFA Stokes imagers. On the one hand, a silicon sensor (CMOS or CCD) has several intrinsic sources of errors, such as the signal-independent additive white Gaussian noise which encapsulates natural phenomena such as thermal noise. Moreover, silicon sensors also undergo the unavoidable and signal-dependent Poisson shot noise, which stems from the discrete nature of electric charge and light. We have seen that the architectural design of the Stokes polarimeters can minimize the impact of these sources of noise, or when in use, figures of merit can be used to quantify the limits in estimation precision of Stokes and polarimetric parameters.

On the other hand, from the polarizing perspective, additional inaccuracy sources are introduced due to the manufacturing process variation of the wire-grid micropolarizers, including optical imperfections of filters and depending on the assembly process as pointed out in Subsection 2.3.4. Thus, each polarizer may exhibit its own optical characteristics, i.e., transmission, diattenuation and polarization analysis orientation [12]. Potential misalignment can also increase the risks of optical crosstalk.

The combination of the above, results in an overall so-called Fixed Pattern Noise (FPN) which can be observed under the same illumination conditions. The FPN that is classified as:

- Photo response non-uniformity (PRNU): where pixels are susceptible to give brighter or darker intensities than others when illuminated with homogeneous light.
- Dark signal non-uniformity (DSNU): where pixels are susceptible to give uneven dark offset when the sensor is exposed to zero irradiation.

The FPN could lead to a spatial variation of numerical values up to 20% over the whole sensing area [81]. An image processing, called spatial calibration procedure, is thus necessary to compensate for these spatial nonuniformities [105]. This step is considered crucial for numerous applications, as uncalibrated camera values could lead to false contrasts or

large errors when computer vision algorithms are applied, e.g. material inspection, shape from polarization, index of refraction retrieval, or illuminant direction estimation.

Besides, as mentioned in chapter 2, one of the drawbacks of PFA Stokes imagers is that they suffer from sparsity, i.e., each pixel senses only one polarization channel and therefore there is a loss in spatial resolution. It introduces Instantaneous Field-Of-View (IFOVs) errors [106] when reconstructing 2D polarization scene information from sparse data. This results in artifacts when the spatial frequency of the image is high compared to the spatial resolution of the camera. Some evolved interpolation methods, like what was done before for the color and spectral domains, have emerged to compensate for these drawbacks [61].

PRNU errors can be counteracted separately from IFOV errors. This chapter focused on the study of existing PRNU calibration methods and applied them using a practical optical acquisition setup and a commercially available PFA camera. The goal of the evaluation is first to compare the existing algorithms in terms of performance (regarding polarimetric errors), and then to investigate both the influence of the dynamic range and the number of polarization angles used for the training data.

Most of the results presented in this chapter have been the subject of one publication that can be found in Appendix A. However, a section relative to the calibration of CPFA cameras is included to generalize the study of calibration to hybrid technologies.

3.2 Calibration techniques

The etymology of the word ‘calibration’ is considered to be derived from the measurement of the caliber of a gun. There is a certain ambiguity in respect to the meaning of the term calibration, therefore, it is worthy to mention that in this manuscript, the use of the term sticks to the International Bureau of Weights and Measures (BIPM) usage; which considers the calibration as “an operation of two steps, that is, the measurement uncertainties and the subsequent use of this information to correct practical measurements” [107]. This idea has been introduced in chapter 1, and when applied to a PFA imager, these two steps refer to the characterization of the analyzer matrix, and the use of this matrix to correct the Stokes estimation. This estimation can be done under two approaches, that in this work; will be referred to as follows:

- *Direct Stokes calibration*: performed out of the estimated analyzer matrix $\mathbf{W}_{\text{est}}^+$, one is able to deduce the calibrated Stokes parameters in each superpixel, and ultimately, assemble calibrated polarimetric images. Such that Equation 1.52 is written as:

$$\mathbf{S} = \mathbf{W}_{\text{est}}^+ \mathbf{I} . \quad (3.1)$$

- *Calibration from the raw image*, which may be useful if subsequent interpolation techniques are to be performed. Such calibration techniques or calibration functions can be defined as:

$$f(\mathbf{I}_j, \mathbf{W}_j) : \mathbf{I}_j \rightarrow \mathbf{I}'_j , \quad (3.2)$$

where the prime symbol denotes the corrected intensity values which already compensate for the nonidealities, therefore one may obtain the Stokes parameters considering an ideal case; by applying Equation 2.6.

The study of such calibration techniques or raw-intensity calibration functions and the optimal setup to characterize the analyzer matrix is the focus of this chapter and will be detailed in the next subsection.

3.2.1 Assumptions & General Procedure

Most of the techniques presented in this chapter have several important assumptions about the sensor, the calibration optical setup, and the statistical behavior of the signal. One assumes that:

- The sensor operates in the linear regime. Most of the calibration techniques also assume that there is no cross-talk effect due to coupling the PFA with the focal-plane array, due to either its position relatively to the microlens array (below or above), or regarding the sensor orientation with the incident light. Moreover, it supposes that the lens configuration is unique and valid only for a given calibration procedure, i.e. f-number and focal length. Any changes in these parameters afterward will deteriorate the calibration result.
- There is no spectral dependency on $\mathbf{W}$, i.e. retardance is flat over the range of wavelengths considered, and the reference polarizer used to generate training Stokes vectors is perfect (no diattenuation, retardance or transmission problems [41]),
- The acquisition during calibration is not corrupted by temporal noise, and that there is no need to apply flat-field procedure to correct training data for residual illuminant spatial deviation.

All the calibration techniques for PFA that will be presented share the same global procedure:

1. Inputting a set of light stimuli (training data) with known Stokes vectors to the polarimeter,
2. For each input Stokes stimuli, capture a series of images and average/add them,
3. Estimate the $\mathbf{A}$ vectors or $\mathbf{W}$ matrices, i.e. the polarization properties at each pixel or super-pixel, by using the pseudo-inverse estimator,
4. Compute the gains and offsets from Step 3., and apply them to correct raw values.

3.2.2 Single-Pixel Calibration

Single-pixel calibration is the way to calibrate each pixel independently, without considering the polarization properties of its neighborhood. Powell and Gruev [81] added the

offset noise d_i in the model in Equation 1.49, to take into account for additive noise during calibration:

$$I_i = \mathbf{A}_i \mathbf{S}_{in} + d_i \quad . \quad (3.3)$$

Then, a calibration function is applied to the pixels as:

$$I'_i = g_i(I_i - d_i) \quad . \quad (3.4)$$

where I'_i is the corrected value and $g_i = \frac{\|\mathbf{A}_{ideal}\|}{\|\mathbf{A}_i\|}$ is the normalized gain, where $\mathbf{A}_{ideal}$ and $\mathbf{A}_i$ vectors are assumed to be co-linear. As stated in Powell and Gruev [81], this assumption implies that only transmission errors can be compensated, the single-pixel calibration does not correct for diattenuation and orientation variations (rotational offset) across the PFA structure. Thus, the single-pixel method yields errors when calculating the angle and degree of polarization.

3.2.3 Super-Pixel Calibration

Super-pixel calibration is a more evolved technique. It uses a relative neighborhood of four pixels to form the super-pixel used in the calibration framework; it compensates for deviations in transmission, diattenuation and orientation. AOLP (Angle Of Linear Polarization) and DOLP (Degree Of Linear Polarization) are much more precise with this technique since an in-common pixel correction is done instead of individual.

Myhre *et al.* [108] use Eq. 2.3 to calibrate their full-Stokes polarimeter. They assume no additive noise in their model. On the contrary, Powell and Gruev [81] include additive offset in their superpixel calibration procedure:

$$\mathbf{I}_j = \mathbf{W}_j \mathbf{S}_{in} + \begin{bmatrix} \mathbf{d}_{j,0^\circ} \\ \mathbf{d}_{j,45^\circ} \\ \mathbf{d}_{j,90^\circ} \\ \mathbf{d}_{j,135^\circ} \end{bmatrix} \quad . \quad (3.5)$$

As for the single-pixel, a calibration function is applied as follows:

$$\mathbf{I}'_j = \mathbf{g}_j(\mathbf{I}_j - \mathbf{d}_j) \quad . \quad (3.6)$$

Where $\mathbf{g}_j = \mathbf{W}_{ideal} \mathbf{W}_j^+$ gathers the four normalized gains recovered by pseudo-inverse operation over $\mathbf{W}_j$.

They found that the super-pixel calibration method reduces reconstruction error, in terms of the Root Mean Square Error (RMSE) for DOLP and AOLP, by a factor of around ten compared to the single-pixel calibration method, along with the correction of diattenuation and orientation.

3.2.4 Adjacent Super-Pixel Calibration

Chen *et al.* [82] performed a complementary strategy to the super-pixel calibration, by adding a computational step at the end, the superpixel algorithm. Once Eq. 3.6 is applied,

every single-pixel is recalibrated as a function of the weighted average of the four overlapped superpixels neighborhood at the pixel position:

$$I_j'' = 1/4[I_j' + I_{jleft}' + I_{jdiag}' + I_{jabove}'] \quad , \quad (3.7)$$

where the terms of average are the calibrated values of the super-pixels located to the left, above and diagonal (adjacent to the latter two), respectively. Authors argue that the error between the acquired DOLP and the ideal value is reduced in range by a factor of ten. In the same way, the error for the AOLP is reduced in range by a factor of four. They also show visually more uniformity in their reconstructed DOLP and AOLP parameters. When applied to a real image, they show that the calibration improves the polarization feature contrasts in both intensity and DOLP images.

3.2.5 Average Analyzer Matrix Calibration

Zhang *et al.* [83] implement the same sequence as Chen *et al.* [82] with the same neighborhood, i.e., by calculating the analyzer matrix and the offsets from training data. However, instead of using an ideal analyzer matrix to produce the multiplicative adjustment of the calibration function, an all-encompassing PFA average matrix is used as the common factor in Eq. 3.6 such that $\mathbf{g}_j = \mathbf{W}_{\text{mean}}\mathbf{W}_j^+$. No analytical criteria are stated for using the mean matrix, but this value is closed to the ideal. When applying the calibration method to a real image, edges become smoother compared to the super-pixel method. Quantitatively, it was found a thousandth order of magnitude reduction of the RMS error between the mean DOLP and the pixel's DOLP. A quantitative comparison with the super-pixel calibration [81] is done using real images, but the authors do not specify how the mean DOLP of an object under study is sampled out of the image.

3.3 Characterization and method comparison

3.3.1 Calibration Setup

The experimental setup used to recover the training and test data is shown in Figure 3.1. The adjustable shutter is controlled by the camera's integration time to modulate the input signal. The light source is a tungsten-halogen lamp and is provided by an Intralux 4000 module. The light passes through a Thorlabs IS200 Ø2" integrating sphere to generate nominally and assumed uniformed and unpolarized light. A 10LP-VIS-B linear polarizer from Newport, whose axis was aligned to the horizon, is considered as the reference polarizer, and it is rotated by an electrically-controlled rotational stage to generate the reference input Stokes vector. The studied PFA camera is assembled by 4D Technology and employs a Sony IMX174 CMOS sensor coupled with a polarization filter array manufactured by the Moxtek company. The photographic objective's focal length equals 12.5 mm and the f-number is set at f/1.4. The camera's digital resolution is set to 12 bits.

With the said setup, the following acquisitions are carried out:

- 6 different intensities, namely, 100%, 50%, 25%, 10%, 5% and 2% (in a way that the 100% maximum intensity equals 75% of the saturation level of the camera) are considered,
- At each intensity, 36 input polarization angles ranging from 0° to 175° with a step of 5° are generated,
- Each couple of intensity/angle image acquisition is averaged over 100 images to generate the final data.

In order to maximize the uniformity of the images, a region of interest is selected so that 300×300 pixels are considered at the center of the sensor area.

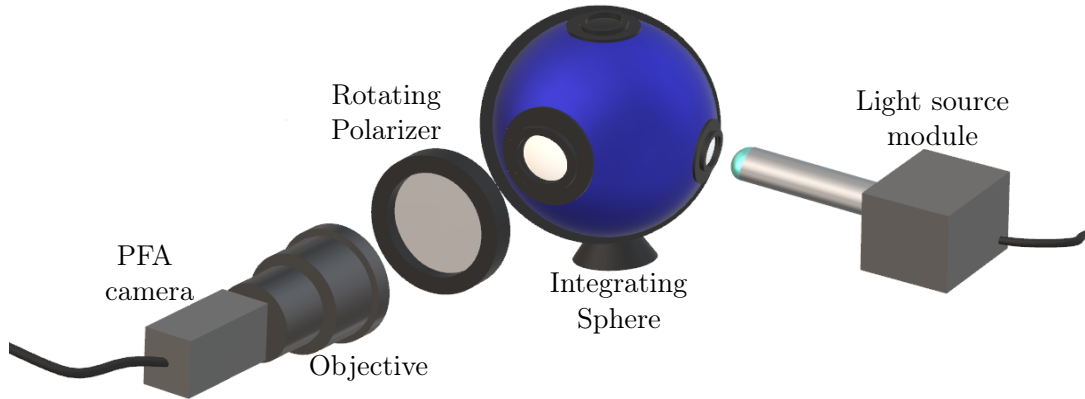


FIGURE 3.1: Acquisition setup used to recover training and test data.

3.3.2 PFA characterization

The characterization of the PFA is carried out by doing an estimation of the analysis vector parameters $\mathbf{A}_i = \begin{bmatrix} a_{0,i} & a_{1,i} & a_{2,i} & 0 \end{bmatrix}$ and the offset noise d_i , at each pixel i . A least-squares solver involving $N = 54$ instances captured from the acquisition setup described previously (9 equally spaced input polarization angles along with 6 intensities) is performed for one pixel, isolating the extended analysis vector in Equation 1.49 as:

$$\begin{pmatrix} \mathbf{A}_i & d_i \end{pmatrix} = \begin{pmatrix} I_{i,1} & \dots & I_{i,N} \end{pmatrix} \begin{pmatrix} \mathbf{S}_{\text{in},1} & \dots & \mathbf{S}_{\text{in},N} \\ 1 & \dots & 1 \end{pmatrix}^+ . \quad (3.8)$$

Where $+$ denotes the pseudo-inverse estimator, and $\mathbf{S}_{\text{in}}$ is the generated reference input Stokes vector, assumed to be uniform across the region of interest.

Optical polarization properties are then derived from the estimated analysis vectors $\mathbf{A}_i$. The extinction ratio ER is calculated by removing the circular components from Equation 1.33 as:

$$ER = \frac{a_{0,i} + \sqrt{a_{1,i}^2 + a_{2,i}^2}}{a_{0,i} - \sqrt{a_{1,i}^2 + a_{2,i}^2}} . \quad (3.9)$$

Likewise, the diattenuation D is obtained from Equation 1.31. The estimated micropolarizer orientation angle θ_i can be deduced from Equation 1.41 as:

$$\theta_i = 0.5 \arctan \left(\frac{a_{2,i}}{a_{1,i}} \right) . \quad (3.10)$$

The polarization nonidealities parameters can be represented by the PFA sensor response to Malus' Law. Figure 3.2 depicts the mean uncalibrated channel responses to Malus' Law; obtained with white linearly polarized light at angles of polarization sweeping the half circle at steps of 5° . It can be seen the peaks of the sinusoids do not occur at their nominal values, as well as an offset in the minima and maxima. The standard deviation is depicted by the shade. It can also be seen that there is a different deviation between the high and low peaks.

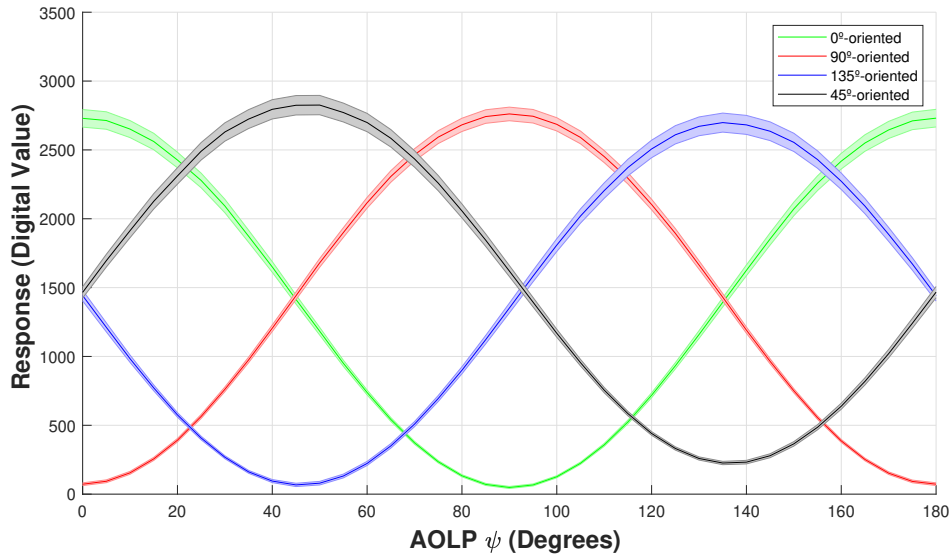


FIGURE 3.2: Uncalibrated mean pixel responses to Malus' Law; separated by orientation channels, obtained with white linearly polarized light at angle of polarization such that: $0 \leq \psi < \pi$. The standard deviation is depicted by the shade.

Besides, the nonidealities are quantitatively represented by the mean values and the standard deviations in each orientation axes (0° , 45° , 90° , and 135°). The assessment makes it possible to see how far the parameters are from their ideal values.

Figure 3.3 depicts the normal-distributed histograms of the PSA's in the characterized sensor area by channel. Besides, Table 3.1 summarizes the mean values and the standard deviation across the characterized sensor area. Results show that most of the polarization orientation axes fluctuate and are displaced from their theoretical values. Diattenuation/extinction ratios also exhibit relatively high spatial variations that could lead to imprecision in the polarization measurement if no correction is applied to raw data.

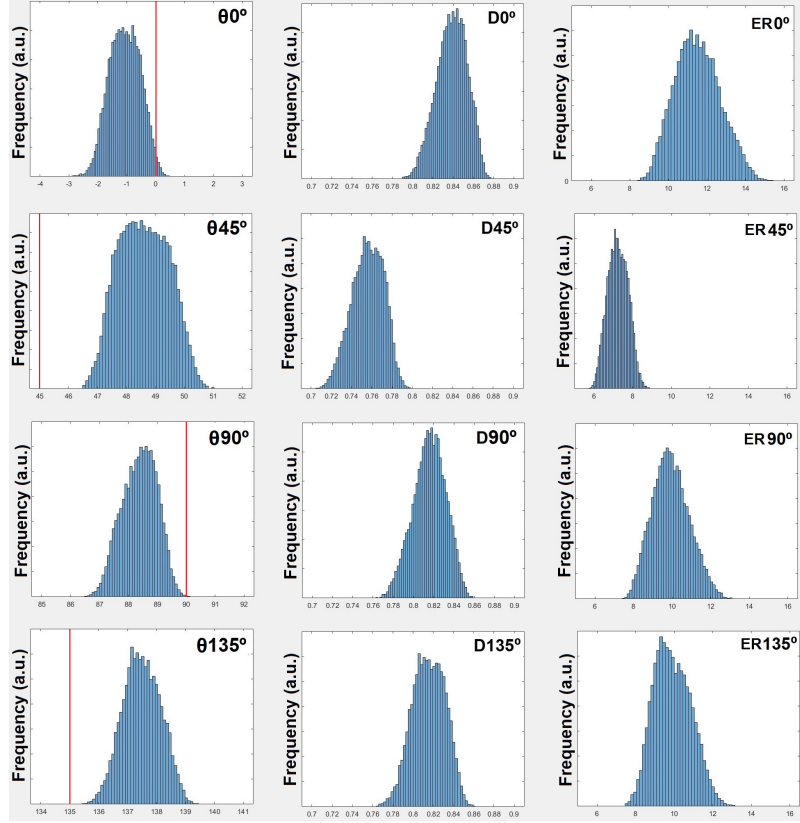


FIGURE 3.3: Count of occurrences of characterized PSA polarizing parameters by orientation channel: filter's orientation angle θ (first column, ideal value indicated with red line), Diattenuation D (second column) and Extinction Ratio ER (third column). These values are estimated over a region of interest of 300×300 pixels at the center of the sensor area.

3.3.3 Calibration method comparison

The four calibration methods presented in Section 3.2 were implemented to evaluate their performance. The test images are acquired when impinging the camera with 20° -oriented fully linearly polarized light at 100%, 50%, and 5% intensity. One can characterize the performance over the dynamic range by means of two metrics, namely; the Root-Mean-Square Error (RMSE) and the Peak Signal to Noise Ratio (PSNR). Results are presented in Table 3.2.

The results show that the single pixel calibration method [81] reduces the error in the S_0 component by an average of 22% but fails to correct S_1 and S_2 , which is in accordance with the inability of the model to correct orientation errors θ as stated by the authors. Conversely, the super-pixel calibration method [81] reduces the RMSE in an average of 95% over the three Stokes components. Adjacent super pixel method [82] yields a ten thousandth order of magnitude reduction of the RMSE with respect to the super-pixel calibration method. However, the adjacent super pixel method implies an interpolation across the super pixel neighborhood. Here the tested image is uniform, so the interpolation step slightly improves the results, acting like a smoothing filter. With a real image containing edges in the polarimetric information, the quality of the correction should decrease. For

Polarization Parameter	Uncalibrated <i>Mean [Std]</i>
θ_0 (deg)	-1.06 [0.53]
θ_{45} (deg)	48.60 [0.85]
θ_{90} (deg)	88.38 [0.60]
θ_{135} (deg)	137.49 [0.65]
D_0	0.839 [0.015]
D_{45}	0.756 [0.016]
D_{90}	0.816 [0.016]
D_{135}	0.815 [0.015]
ER_0	11.51 [1.172]
ER_{45}	7.23 [0.526]
ER_{90}	9.93 [0.955]
ER_{135}	9.91 [0.962]

TABLE 3.1: Mean values and standard deviation (square brackets) of the estimated polarimetric properties: orientation angle θ , diattenuation D and extinction ratio ER . These values are estimated over a region of interest of 300×300 pixels at the center of the sensor area.

the average analyzer matrix method [83], it can be seen a reduction of the RMSE in an average of only 4% over the three Stokes components. This might be due to the replacement of an ideal analyzer matrix by an uncalibrated mean analyzer matrix, which could carry the errors in the parameters.

Super-pixel calibration method is a well-established method. The basic concept of superpixel is a starting point for the other methods. Besides, the Stokes vectors obtained by Super-pixel raw-intensity calibration are completely equivalent to the direct Stokes calibration, as the replacement of Equation 3.6 in Equation 2.5 turns out in Equation 1.52, if one considers the intensity measurement vector with the offset subtracted, but with the advantage of having access to the calibrated raw image. Derived methods do not necessarily have significantly better results.

To illustrate the super-pixel method performance, Figure 3.4 shows pseudo-color images representing the DOLP and AOLP before and after the super-pixel calibration applied to the test image with input polarization light at 0° and 100% intensity.

When the image is uncalibrated, the DOLP has a mean value significantly lower than 1. The color scale allows to intuitively visualize the heterogeneity of the values in the flat field. The mean AOLP is improved by shifting near the ideal value, and its standard deviation is improved by a factor of 25.

Figure 3.5 depicts the mean superpixel calibrated channel responses to Malus' Law; obtained with white linearly polarized light at angles of polarization sweeping the half circle at steps of 5° . It can be seen the peaks of the sinusoids do occur at their nominal values, as well as no offset in the minima and maxima. The standard deviation is depicted

	Uncalibrated	Single-Pixel [81]	Super-Pixel [81]	Adj. S-P [82]	Av. Matrix [83]
100% Light Intensity					
S_0	0.1203/18.39	0.094/20.53	0.0005/65.68	0.0004/67.93	0.1183/18.53
S_1	0.1147/18.80	0.113/18.94	0.0025/52.02	0.0024/52.25	0.1102/19.15
S_2	0.1132/18.92	0.1411/17.01	0.001/61.23	0.001/63.88	0.1082/19.31
50% Light Intensity					
S_0	0.1277/17.88	0.1011/19.91	0.0064/43.83	0.0064/43.86	0.1257/18.07
S_1	0.1057/19.52	0.1036/19.70	0.0088/41.07	0.0088/41.13	0.1006/19.95
S_2	0.1053/19.56	0.133/17.52	0.0102/39.81	0.0102/39.86	0.0996/20.03
5% Light Intensity					
S_0	0.1231/18.20	0.0965/20.31	0.0033/49.54	0.0026/51.67	0.1209/18.35
S_1	0.1093/19.23	0.1073/19.391	0.006/44.401	0.0048/46.39	0.1043/19.63
S_2	0.1118/19.03	0.1395/17.111	0.0048/46.33	0.0031/50.08	0.1064/19.45

TABLE 3.2: Mean Root-Mean-Square Error (RMSE)/ Mean Peak Signal to Noise Ratio (PSNR) in terms of the Stokes vector components S_0 , S_1 and S_2 for the uncalibrated and calibrated responses of a 20° -oriented fully polarized light capture in a region of 300×300 pixels.

by the shade. It can be seen that the standard deviation has been highly reduced and evened out, in respect to the uncalibrated case depicted in Figure 3.2.

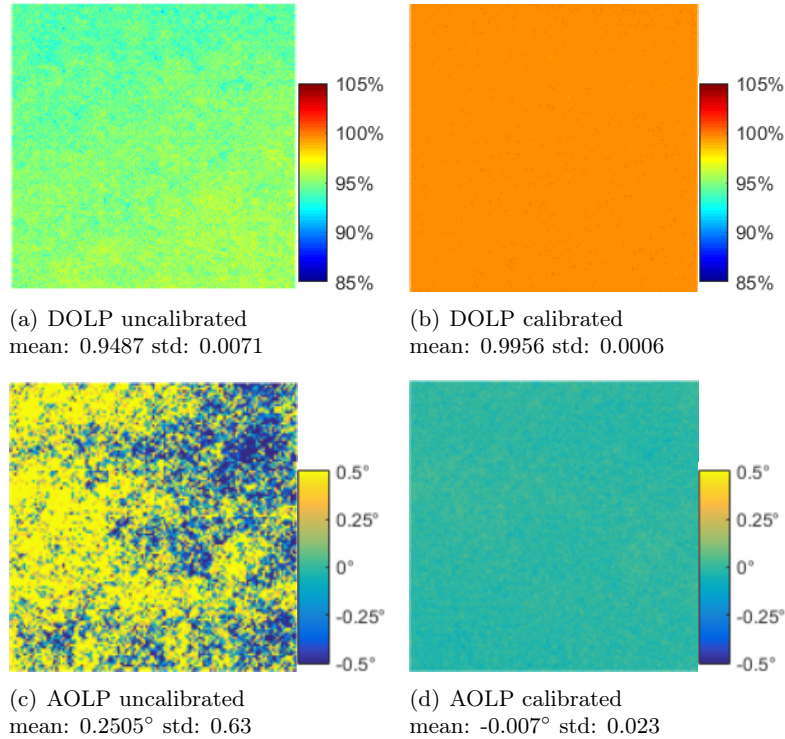


FIGURE 3.4: Zero-oriented fully-polarized light capture and parameter estimations. The region of interest is an area of 300×300 pixels in the center of the sensor.

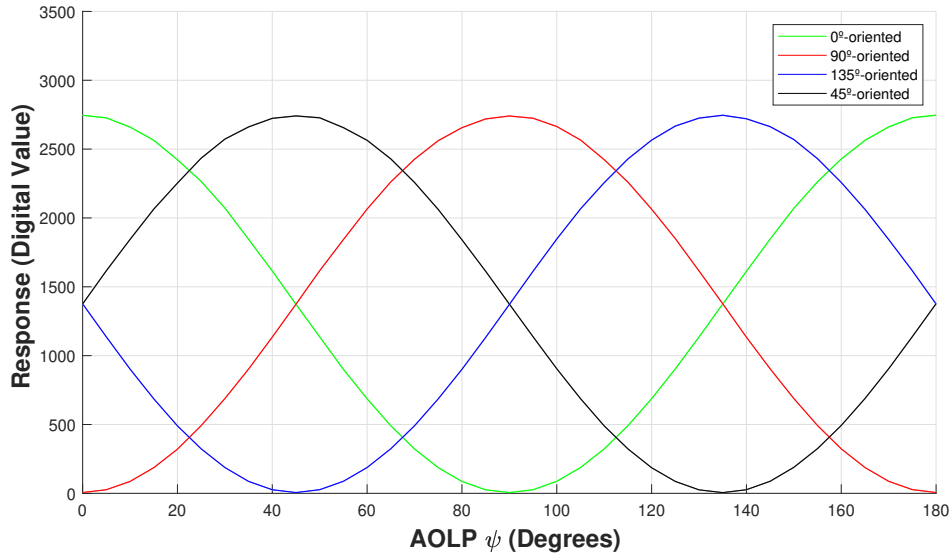


FIGURE 3.5: Superpixel calibrated mean pixel responses to Malus' Law; separated by orientation channels, obtained with white linearly polarized light at angle of polarization such that: $0 \leq \psi < \pi$. The standard deviation is depicted by the shade.

3.4 Characterization of CPFA Stokes imagers

The development of the CPFA sensor introduced in Chapter 2, can be thought of as a technological convergence, i.e, is the tendency for technologies that were originally unrelated (in this case color and polarization), to share common elements of digital electronics, hardware and software. When using the Color-Polarization Filter Array (CPFA) imager architecture, one has to take into account the intended purpose given by the user to define a polarimetric calibration pipeline and estimate the estimation precision.

One can consider a CPFA imager simply as a compact camera that can be used alternatively as a color camera as well as a polarimetric imager, but also as a spectral and polarization imaging system. Spectral and Polarization Imaging (SPI) is an emerging sensing method that combines the acquisition of both spectral and polarization information of a scene, and it could be used in various applications like appearance characterization, reflectance property estimation, diffuse specular component separation, among others [90]. For example, Lapray *et al.* [109] developed a spectro-polarimetric imaging system in the visible and near-infrared regions. In this work, the polarimetric feature is achieved using a Division-of-Time architecture and the calibration procedure is done separating radiometric, spectral and polarimetric properties. Qiu *et al.* [110] proposed a simultaneous color/polarization demosaicing algorithm for color PFA cameras and performed the calibration of polarization features following the calibration method pointed out in [85].

This section deals with the study of the polarimetric calibration in the spectralpolarization case, framed in the color filter array depicted in Chapter 2. Including the determination of the estimation precision when the camera is to be used as a polarimetric imaging tool and how is it compared with respect to the monochrome case; when additive and

Poisson noise are the main source of inaccuracies.

3.4.1 CPFA polarimetric calibration pipeline

The Quad Bayer-Chun CPFA imager is characterized by a primitive cell made up of a 4-by-4 superpixel. From the Spectral-Polarization perspective, the 4-by-4 superpixel can be divided up to 12 channels (if the two diagonal green channels are considered to be unified). The PFA Linear imaging model of subsection 2.3.3, assumes that either the incident illumination is uniform across the superpixel or that all the constituent pixels are co-located [81]. The intrinsic arrangement Quad Bayer-Chun fulfills this characteristic, and therefore the polarimetric calibration, either direct Stokes calibration or calibration from the raw image, can be applied without any rearrangement on the mosaicked image (with a 4-by-4 superpixel unit cell), or in the subsampled monochromatic intensity images (2-by-2 superpixel unit cell). Figure 3.6 shows a block diagram of a subsampling calibration pipeline for a Quad-bayer CPFA imager under the sensor alignment that will be considered henceforth, that is, color alignment BGGR and subpolar alignment: 90-135-45-0, until the 12 monochromatic θ -oriented channels.

Conversely, in a CPFA imager featuring a Tu Bayer/Chun [88], the calibration can not be done directly on the mosaicked image (with a 4-by-4 superpixel unit cell). Instead, the image must be rearranged following a nearest-neighbor same-color subsampling and then carry out the calibration process, either direct Stokes calibration or calibration from the raw image. As a comparison, the Quad Bayer-Chun CPFA arrangement exhibits an oversampling of polarization channels as compared to the color channels, whereas, Tu Bayer/Chun it is the inverse, i.e. color channels are oversampled in comparison to the polarization channels. Figure 3.7 shows a block diagram of a subsampling calibration pipeline for a Tu Bayer/Chun CPFA imager under the same BGGR sensor alignment, until the 12 monochromatic θ -oriented channels. This approach is done under the assumption that the incident illumination is uniform across the 3-by-3 pixels vicinity [77].

3.4.2 Implementation of CPFA characterization

For this study, the optical setup is the same as shown in Figure 3.1, replicated for an onchip Quad-bayer CPFA camera assembled by Lucid Vision Labs featuring an on-chip SONY IMX250MYR sensor [89]. The adjustable shutter is substituted by the camera's integration time to modulate the input signal. The light source passes through a Thorlabs IS200 Ø2" integrating sphere to remove any polarization signature and produce a uniform light beam. A 10LP-VIS-B linear polarizer mounted on a rotational stage is used as the reference polarizer. The acquisition was performed as pointed out in Subsection 4.3.1.

Table 3.3 gathers the mean values and spatial standard deviations of the estimated optical polarization properties: extinction ratio ER and orientation angle θ for each spectral band separately throughout the characterized sensor's region. By examining the extinction ratio along the same color channel, it can be seen that they keep the same order of magnitude. By contrast, if compared along different color channels in same-orientated pixels,

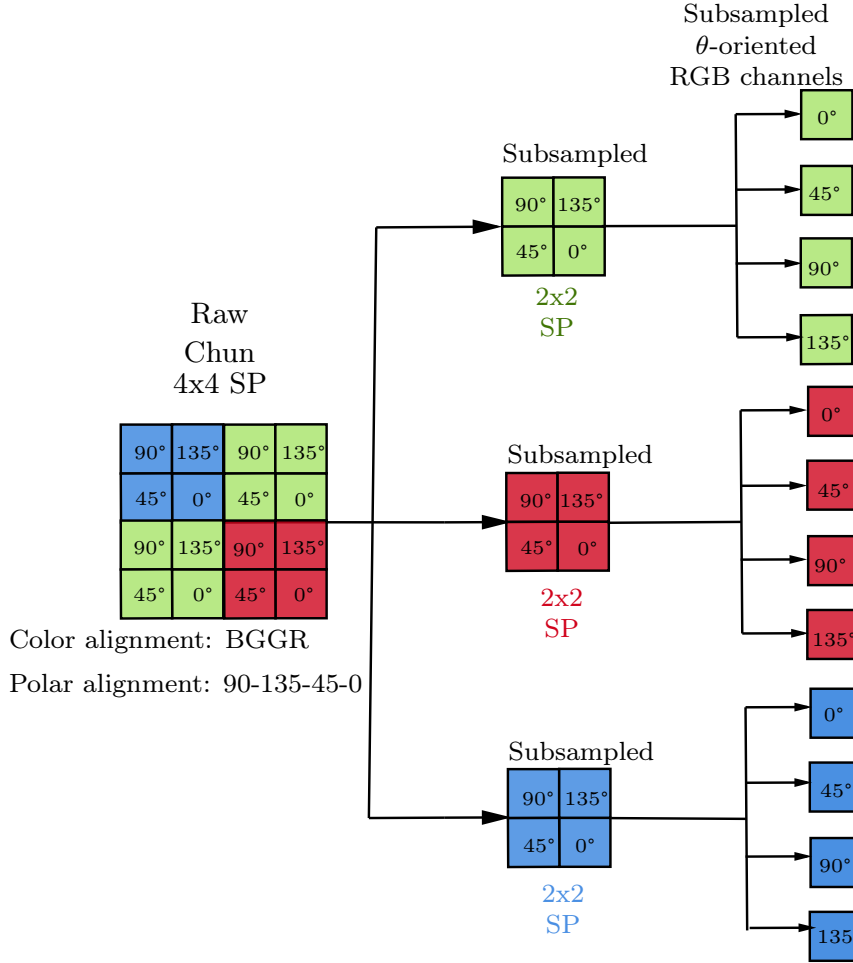


FIGURE 3.6: Block diagram of the subsampling calibration pipeline for a Quad-bayer CPFA imager. The Superpixel polarimetric calibration can be performed either in the raw image (defined by a 4-by-4 super pixel) or separately in the subsampled monochromatic raw intensity images (defined by a 2-by-2 super pixel).

the average of the green channel exhibits roughly twice the average of both the red and the blue channels. This can be attributed to the fact that this property, is limited by the dynamic range and the noise level of the pixel sensor as it is obtained the ratio between the maximum and the minimum intensity signals, and they turn out different among the three color channels for a wideband uniform target.

3.4.3 Estimation precision in Spectral-polarization imagers

This subsection is focused on scenarios where detection noise is the dominant source of inaccuracies, assuming that the spatial nonuniformities have been balanced out by the calibration. As a result, it must be established a noise model that bring together both polarimetric and spectral properties to characterize the dispersion error in the Stokes parameters, and how they behave among the different color channels. Characterization of errors due to noise has been widely studied encompassing all the polarimetric architectures as pointed out in subsection 2.2.6. Moreover, the study has been confined to monochrome

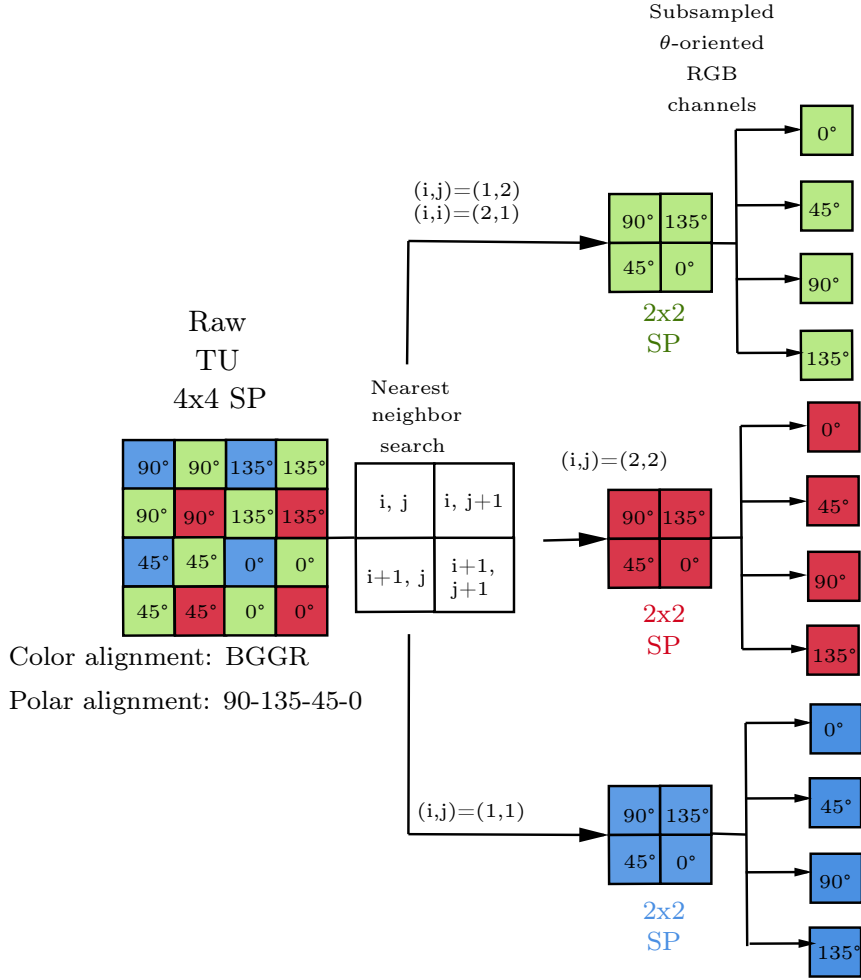


FIGURE 3.7: Block diagram of the subsampling calibration pipeline for a Tu Bayer/Chun [88] CPFA imager. The Superpixel polarimetric calibration must be performed after nearest-neighbor subsampling to synthesize the same-color channels (defined by a 2-by-2 super pixel). The indices (i,j) are applicable for the four color channels.

PFA cameras [54, 55], notably, for this architecture, it has been discussed that in the emerging generation of high extinction ratio cameras (e.g. more than 100), the temporal noise is dominant. Consequently, the relevance of this topic in emerging microgrid technologies is what motivates this discussion, generalizing the study to the case of CPFA imagers.

In the classical color imaging, the noise has been also characterized with the purpose of being counteracted by a process known as noise reduction or denoising and, the order to be performed in respect to the chromatic interpolation step (also known as demosaicing). For example, some approaches perform joint demosaicing and denoising, or carry out denoising prior to demosaicing [111, 112, 113]. Moreover, in the same color-imaging context, the noise models are based on different assumptions. For instance, a Gaussian model with signal-dependent and non-stationary standard deviation [112] or a signal independent Gaussian model with constant variance in each color channel, [114], to name a few.

To define the estimation precision in a CPFA imager, firstly, let us consider the expression of the EVW_{add} in an ideal CPFA camera. As the AWGN is present with zero

Polarization Parameter	Red Channel <i>Mean [Std]</i>	Green Channel <i>Mean [Std]</i>	Blue Channel <i>Mean [Std]</i>
ER_0	61.41 [3.95]	102.75 [11.17]	46.09 [3.09]
ER_{45}	54.78 [3.52]	101.55 [12.34]	49.85 [3.33]
ER_{90}	62.19 [3.42]	109.03 [9.15]	54.45 [4.07]
ER_{135}	81.53 [5.72]	141.31 [18.51]	58.92 [5.29]

TABLE 3.3: Mean values and standard deviation (square brackets) of the estimated optical polarization properties: Extinction ratio ER for the red, green, and blue channels. These values are estimated over a region of interest of 700×700 pixels at the center of the sensor area.

irradiation and will generally depend on other parameters, especially exposure time and ambient temperature, the dark noise is independent of the color channel and polarization-state of the input signal. The additive or background noise model for monochrome cameras previously studied assumes that the dispersion among the pixel array is stationary [25, 41], that is, the temporal variance σ^2 is the same for all pixels and therefore in the superpixel whatever sensor alignment in the sampling. This assumption holds for the CPFA camera. Therefore, in the ideal case, the autocovariance matrix of dark noise in the three color channels will obey [9]:

$$\Gamma_{r,g,b}^{add} = \sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \quad (3.11)$$

And therefore the Equally Weighted Variance can be obtained by applying Equation 2.1 which gives:

$$EWV_{r,g,b}^{add} = 5\sigma^2. \quad (3.12)$$

Conversely, to model the expression of the Equally Weighted Variance in the presence of only Poisson noise in a CPFA camera, one focuses on the measured digital values since it is the value that the camera outputs, rather than the number of photons or electrons. Assuming that the measured intensities are statistically independent and Poisson random variables, the expected value or mean intensity vector can be written as:

$$\langle \mathbf{I}_{r,g,b} \rangle = \begin{bmatrix} \langle I_0 \rangle \\ \langle I_{45} \rangle \\ \langle I_{90} \rangle \\ \langle I_{135} \rangle \end{bmatrix}. \quad (3.13)$$

Where the angle brackets $\langle \cdot \rangle$ denote the mean value operator, and $\langle \mathbf{I} \rangle$ represents a fixed vector whose elements are the mean value of its components, that is, the four intensities in the superpixel. Due to the statistical properties of the Poisson distribution, the variance

of one such variable equals its mean, in a way that Equation 3.13 can be written as:

$$\langle \mathbf{I}_{r,g,b} \rangle = VAR[\mathbf{I}_{r,g,b}] = \begin{bmatrix} VAR[I_0] \\ VAR[I_{45}] \\ VAR[I_{90}] \\ VAR[I_{135}] \end{bmatrix}. \quad (3.14)$$

In the ideal case, it has been shown that, the expression of the covariance matrix in the presence of Poisson noise equals to [9]:

$$\Gamma^{Poi} = \frac{1}{2} \begin{bmatrix} S_0 & S_1 & S_2 \\ S_1 & 2S_0 & 0 \\ S_2 & 0 & 2S_0 \end{bmatrix}. \quad (3.15)$$

And therefore the Equally Weighted Variance can be obtained by applying Equation 2.1 which gives:

$$EWW^{poi} = \frac{5}{2} S_0. \quad (3.16)$$

It can be seen that in the ideal case, the presence of Poisson noise, the EWWV depends only on the total illumination intensity S_0 , which is not a polarimetric property of light. In a CPFA imager, the number of electrons and therefore the measured digital number is different for each spectral channel as color filters narrow the input bandwidth and the quantum efficiency is wavelength-dependent. Thus, the intensity of each channel must be modeled as a function of the spectral properties of the input light and the window size of the bandpass filters that determine the quantum efficiency.

Let $H(\lambda)$ be the spectral density of the input light, $\eta_{r,g,b}(\lambda)$ the quantum efficiency in the corresponding RGB color channels, then, the spectral density color filter's output $G_{r,g,b}(\lambda)$, defined by the resulting bandwidth with cutoff frequencies λ_1 and λ_2 , is given by the convolution in the direct domain or the product in the frequency domain between the first two parameters. Figure 3.8 shows a block diagram of the relation among these magnitudes. The response caused by light of different wavelengths are linearly superimposed (depicted as integration) between $\lambda \in [\lambda_1, \lambda_2]$.

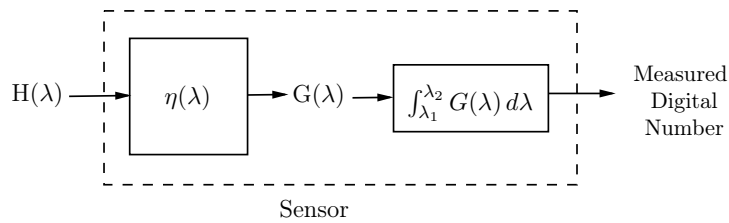


FIGURE 3.8: Block diagram of the frequency response $G_{r,g,b}(\lambda)$ resulting from the product between the spectral density of the input light $H(\lambda)$ and the quantum efficiency $\eta_{r,g,b}(\lambda)$ in each color channel. The response caused by light of different wavelengths are linearly superimposed (depicted as integration) between $\lambda \in [\lambda_1, \lambda_2]$.

Based on the above, Equation 3.16 can be generalized as follows:

$$EVW_{r,g,b}^{poi} = \frac{5}{2} \int \eta_{r,g,b}(\lambda) H(\lambda) d\lambda . \quad (3.17)$$

3.5 Calibration setup optimization

This section seeks to optimize the number of acquisitions during the characterization process, being a crucial step as the camera has to be recalibrated several times as the f-number or the focal length is changed over time. A wide variety of calibration methods employ calibration setups which seem to be oversized or not well optimized relatively to these two parameters. This investigation could be useful to make the calibration setup lighter in terms of number of acquisitions.

Here, the effect of varying the training data configuration over the calibration pipeline is investigated. Two criteria are individually tested: namely, the dynamic range of input data and the number of polarization angles used to train the calibration algorithm.

The super-pixel calibration from Powell and Gruev [81] is applied to the evaluation as it is the most generic and in-common starting method employed in all previously reviewed methods.

3.5.1 Impact of data dynamic range

The analysis of polarization in a scene with a camera could exhibit pixel regions that sense low irradiance or high polarization degree [115]. Thus, one can envisage to use several images with different dynamic ranges of intensity during characterization and calibration.

To see how dynamic range of training images impacts the calibration result, the characterization and train the model using 6 scenarios regarding the Dynamic Range (DR) magnitude were carried out such that: all intensity images combined, only 100%, combination of 100%|50%, 100%|50%|25%, 100%|50%|25%|10%, 100%|50%|25%|10%|5% intensity levels. 9 equally-spaced input polarization angles were used for each scenario in order not to be affected by the angle selection criteria, which will be tested in the next subsection in this evaluation step. Test of the calibration is done using only one image at one specific angle (10°) which is not in the training set, and with 6 different intensities.

Table 3.4 examines the PSNR reconstruction of the Stokes vector components after applying the calibration over the scenarios. It can be seen that the error has different trends over the three components, so it is not obvious that using a high dynamic range increases the performance of the calibration function. Different analyses could be done by looking at the results:

- The three Stokes parameters are not affected similarly by the DR enhancement of training data, the S_0 component is the most affected.
- Using a great variety of DR increases the global performance by looking at the mean values over the tested images. By evaluating an equally-weighted average over the three components, the 2-DR image scenario gives the highest PSNR, whereas the

(a) S_0								
	100%	50%	25%	10%	5%	2%	Mean	[Std]
100%	54.75	40.76	51.46	46.62	45.20	28.08	44.48	[9.40]
100% 50%	60.73	46.74	57.57	52.62	51.20	34.05	50.48	[9.42]
100% 50% 25%	57.02	47.95	58.61	51.31	50.81	34.34	50.01	[8.66]
100% 50% 25% 10%	55.08	48.82	58.23	50.45	50.39	34.52	49.58	[8.17]
100% 50% 25% 10% 5%	54.37	49.21	57.85	50.10	50.20	34.60	49.39	[7.95]
All DR	50.43	52.55	53.78	47.73	48.45	35.14	48.01	[6.71]

(b) S_1								
	100%	50%	25%	10%	5%	2%	Mean	[Std]
100%	55.36	41.46	44.11	48.51	46.48	33.53	44.91	[7.30]
100% 50%	48.63	44.00	47.56	51.20	47.87	34.41	45.61	[5.96]
100% 50% 25%	48.05	44.38	48.10	51.33	47.91	34.52	45.71	[5.91]
100% 50% 25% 10%	48.03	44.39	48.13	51.36	47.93	34.53	45.73	[5.91]
100% 50% 25% 10% 5%	48.01	44.40	48.15	51.37	47.93	34.53	45.73	[5.91]
All DR	46.79	45.35	49.50	51.28	47.86	34.80	45.93	[5.83]

(c) S_2								
	100%	50%	25%	10%	5%	2%	Mean	[Std]
100%	55.90	47.71	52.42	50.79	47.13	39.16	48.85	[5.73]
100% 50%	59.49	49.36	54.22	49.95	46.56	39.60	49.86	[6.76]
100% 50% 25%	60.48	49.93	54.69	49.66	46.37	39.75	50.15	[7.07]
100% 50% 25% 10%	60.79	50.11	54.85	49.59	46.32	39.79	50.24	[7.18]
100% 50% 25% 10% 5%	60.91	50.17	54.92	49.57	46.31	39.80	50.28	[7.21]
All DR	61.63	51.90	55.55	48.69	45.74	40.15	50.61	[7.54]

TABLE 3.4: Results of the Peak Signal to Noise Ratio (PSNR) calculated over the S_0 , S_1 , and S_2 parameters for each scenario. The dynamic range effect of training (row) and testing (column) data is evaluated through these tables. A single capture with an angle of 10° is used as test image. Best values are highlighted in green, and worst values in red.

standard deviation is inversely proportional to the number of realizations. Intermediate DR images in the set insignificantly increase the calibration performance.

- If dealing with polarization signatures that are mostly close to the noise level of the sensor, it is preferable to include low DR in the training set.

To summarize, adding more than just one DR image to the training setup will enhance globally and significantly the result of calibration, which seems intuitive. Using only 2 different DR seems to be enough to increase significantly the results compared to training with only 100% intensity images, adding more is not judicious.

3.5.2 Impact of training angle selection

To evaluate the importance of the number of input polarization angle acquisitions made for the characterization, the super-pixel calibration is applied over a set of images captured when the camera is illuminated with uniform polarized light at 100% of intensity, and by varying both the number of polarization angles for training and the polarization angle for test. To this end, we first train and characterize the analysis vectors using 6 scenarios: 36, 18, 9, 5, 4, and 3 equally distributed angles images at 6 different intensities. The latter setup involved the solved Equation 3.8, which is under-determined as the number of instances is not equals to the number of unknowns. Thus, the said scenario is possible

assuming that the dark offsets are small compared to the dynamic range of the camera (12-bit intensity). Dark offset could also be determined experimentally [39].

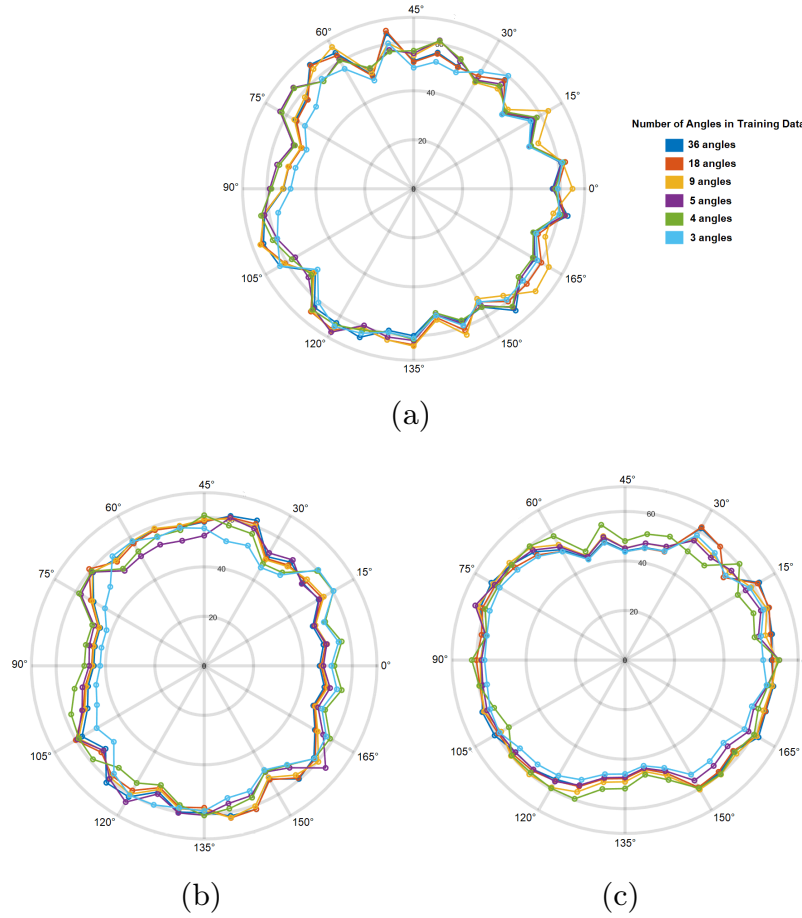


FIGURE 3.9: Peak Signal to Noise Ratio (PSNR) of Stokes parameters after calibration as a function of the number of angles used to calculate the training data. Subfigures (a), (b), and (c) depict the signal level in the parameters S_0 , S_1 , and S_2 respectively. Each color represent a different setup and the radius corresponds to the PSNR. Test Images are swept from 0° to 175° with a step of 5° .

Once the scenarios are trained, we apply the correction on several single test images corresponding to 36 different input polarization angles (from 0° to 175° with a step of 5°) at 100% of intensity. Then, we measure the errors relatively to the Stokes reference input light for each scenario and tested angles.

Table 3.5 examines the Peak Signal-to-Noise Ratio (PSNR) in terms of the Stokes vector parameters, compiling the mean values and standard deviation (square brackets) evaluated in the above scenarios. The means have different levels in the vector components, and they have a peak in the 9-angles setup in both S_0 and S_2 components. However, it can be seen that the standard deviation is also higher at this point, which counteracts its ranking. Conversely, the use of 4 angles yields only 2% less in S_0 but with a lowest standard deviation, along with better values in S_1 and the lowest standard deviation in S_2 . Figure 3.9 breaks down the said PSNR behavior when sweeping from 0° to 175° . It

Training Data	S0 <i>Mean [Std]</i>	S1 <i>Mean [Std]</i>	S2 <i>Mean [Std]</i>
36 angles	57.46 [5.36]	54.35 [5.59]	55.85 [6.10]
18 angles	57.91 [5.47]	54.44 [5.04]	55.67 [6.06]
9 angles	58.69 [5.48]	54.49 [5.04]	56.28 [5.35]
5 angles	57.83 [4.41]	54.05 [4.79]	54.78 [5.13]
4 angles	57.66 [4.28]	55.03 [3.97]	56.12 [4.40]
3 angles	55.72 [5.09]	52.62 [5.56]	53.55 [5.59]

TABLE 3.5: Peak Signal to Noise Ratio (PSNR) mean values and standard deviation (square brackets) in terms of S_0 , S_1 , and S_2 parameters over testing data ranging from 0° to 175° with a step of 5° as a function of training data calculated out of different number of angles (36, 18, 9, 5, 4 and 3 angles) and 6 intensities levels. Best values are highlighted in green, and worst values in red.

can be seen that the signal level is not uniform with respect to the input polarization angle, nevertheless, the fluctuations roughly keep the same shape among scenarios. In this respect, it can be seen that there is a trade-off between this simplification and the output error when cutting down the amount of training measurements.

A simpler setup to characterize each pixel individually by using measurements in 4 different angles instead of 54 measurements (9 angles and 6 intensity levels) as the training data used in the above standalone evaluation, has been proposed [73]. The said method put forward a single-pixel calibration approach with a simpler calibration setup than prior calibration methods [81, 83, 82, 75]. They use only four measurements to characterize each pixel individually and recover the incident intensity, the orientation axis of analysis, and the diattenuation parameters. Under this approach, a motorized rotational stage is no longer needed, as an angle-graduated mount is a practical solution. This choice is in accordance with our results, as we found that it is not necessary to use more than 4 input angles for the calibration.

By applying the same criteria as in the calibration method comparison in Section 3.3.3, it has been verified that characterizing the analysis vectors with 4 angles with one intensity delivered a 93% RMSE reduction averaged over the three Stokes components with respect to the uncalibrated data. Comparatively, the 9 angles and 6 intensity level scenario yields a relatively small improvement (0.67% in RMSE reduction). The difference is attributed to the fact that the latter option considers more than one dynamic range.

3.6 Conclusion

In this chapter, existing calibration algorithms have been reviewed and evaluated using a practical implementation applied to a commercially available monochrome PFA camera. The camera's polarization optical parameters were characterized and methods were applied to the data using a uniform linearly polarized light. The results obtained show that the

super-pixel method performs well, and that any other methods that are derived from it brings no significant enhancement.

The calibration setup optimization, has been discussed, considering the impact of data dynamic range and the impact of training polarization angle selection. This study considers separately the influence of above-mentioned two parameters on the Signal-to-Noise Ratio of the Stokes vector components. In this way, it can help to arrange the optimal setup when calibrating a PFA camera leading to simpler design, with less acquisition needed.

The results show that using more than four angle realizations does not significantly improve the PSNR in the Stokes vector components. But it has been demonstrated that using two different dynamic range images improves significantly the calibration compared to using one. To summarize, a calibration using training data that consist in four polarization angles and two dynamic range realizations is a good compromise in terms of PSNR, while simplifying the calibration setup described in Powell and Gruev [81]. In the next chapter, the impact of the optical path in case of using objective lenses with different configurations, along with the impact of the uniformity of the light used to generate the training data, will be studied.

Chapter 4

Impact of optical path in the calibration of polarizer filter array Stokes imagers

4.1 Introduction

In most applications, the DoFP polarimeter is used in conjunction with an objective lens whose optical parameters (focal length and aperture) are set depending on the target to render a neat image. As a result, this variable optics stage may represent an additional and variable source of optical crosstalk. Most of the calibration methods described in the literature assume that light rays are at normal incidence to the polarimeters, hence with a negligible polarimetric crosstalk. It is only an appropriate assumption for large focal-length or small aperture lenses. This is the case for the work by Hsu *et al.* [65], where they designed and calibrated a full Stokes polarimeter using an array of elliptical polarizers.

York *et al.* [85] characterized the effect of optical path of the incident light. They demonstrated that the collimation of light has a huge importance on the AOLP and DOLP estimation precision. It appears that wide field-of-views (or short focal lengths), depicted in Figure 4.1, as well as greater iris apertures (smaller f-numbers), depicted in Figure 4.2, leading to more divergent light, finally produces a noticeable polarimetric performance degradation. The same behavior has also been depicted by Myhre *et al.* [64] for a liquid crystal PFA polarimeter. Tauc *et al.* [116] studied the impact of the reflections between the reference polarizer and the pixel polarizer in the short-wave infrared, so the reflective properties are in this case accounted in the calibration.

In Chapter 3, the calibration method and setup optimization from the imaging polarimeter side have been established for a fixed objective lens setup. It has been discussed that the analyzer matrix depends on the nonidealities and the spatial nonuniformities of the micropolarizers (in transmittance, diattenuation, and orientation).

The above process of polarimetric calibration requires the micro-polarizers' PSA characterization, which accounts for both photometric crosstalk and polarimetric defects. Therefore, when the sensor is impinged with fully collimated light, the PSA's polarizing properties must be characterized for each incident angle and consequently for each objective's f-number that the polarimeter uses [81, 73] and in some instances for each objective lens [117].

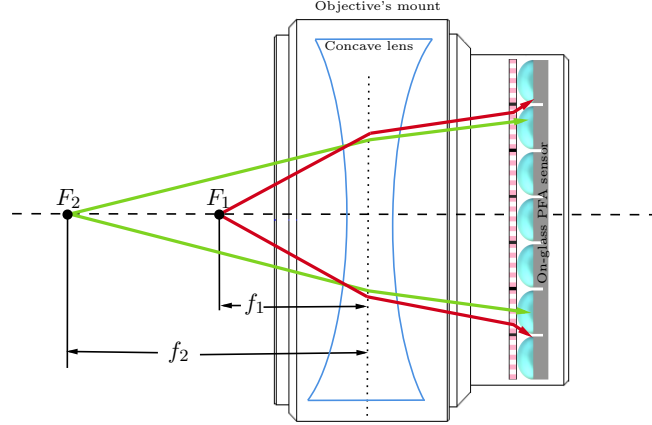


FIGURE 4.1: Cross-sectional representation of the objective's focal length set by two focal points F_1 and F_2 . The longer the focal length (f_2), the narrower the angle of view and the lower the probability of optical crosstalk. The shorter the focal length (f_1), the wider the angle of view and the higher the opportunity of crosstalk.

This chapter is intended to study the dependency over the incoming light geometry as the objective's optical parameters are varied, relative to broad-spectrum and uncollimated light, as a typical scene contains a combination of these parameters. The study will be carried out by using two commercially available PFA cameras with different sensor's process assembly structures discussed in subsection 2.3.2. Then, a generic and calibration method is proposed to simplify the calibration process of the DoFP. This may be useful in cases where the same scene must be captured with multiple optical configurations and be merged by an image processing algorithm[118] to form the final image.

4.2 Simulated numerical nonideality error model

The measured analysis vector components determine the estimation of the PSA's polarizing parameters, namely, orientation angle θ , and extinction ratio ER , whose nonidealities are calibrated as shown in the previous chapter. To illustrate the impact of the errors in the estimated analysis matrix, let us consider a particular case where the analysis vector deviates from the ideal nominal values as a function of a numerical additive term ξ , that encapsulates the nonidealities in the maximum and minimum transmittances as:

$$\frac{1}{2} \begin{cases} -0.1 < \xi \leq 0, & \forall \mathbf{A}_{0^\circ}, \mathbf{A}_{45^\circ} \\ 0 < \xi \leq 0.1, & \forall \mathbf{A}_{90^\circ}, \mathbf{A}_{135^\circ}, \end{cases} \quad (4.1)$$

whose value is applied to the second and third analysis vector components.

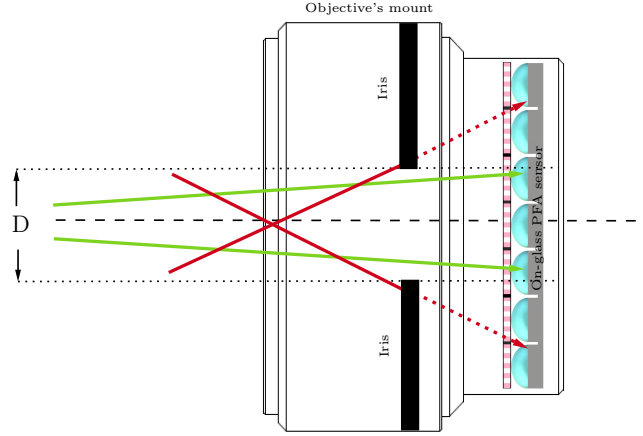


FIGURE 4.2: Cross-sectional representation of the objective's f-number, which is inversely proportional to diameter of the iris' aperture D . The shorter the diameter, the narrower the range of oblique angles that impinge the sensor. The longer the diameter, rays such as the red arrows; would pass through, and the higher the probability of optical crosstalk

4.2.1 Estimated micropolarizer orientation angle

The shift in the estimation of orientation axis θ can be represented by combining Equations 2.4 and 3.10 as a function of the above additive term, ξ as follows:

$$\theta = 0.5 \arctan \left(\frac{a_2 + \xi}{a_1 + \xi} \right) . \quad (4.2)$$

Let us consider the variation of the orientation angle in the 0° -oriented channel, where $\mathbf{A}_{0^\circ} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}$. Figure 4.3 shows the simulated result.

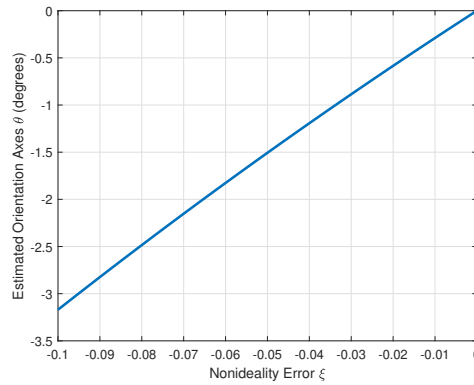


FIGURE 4.3: Angle error in the estimated pixel's orientation angle 0° as a function of an additive nonideality error ξ .

It can be seen that the estimated orientation angle value can be shifted in respect to a referential angle as the error increases. The difference of the error ξ among the pixels in a superpixel yields an uneven angular distribution and leads to a source of inaccuracy in the subsequent estimated parameters. Nevertheless, it has been determined that increasing the diattenuation/extinction ratio makes it possible to relax the manufacturing constraints on

the orientation precision of micropolarizers [54], thus the analysis will be focused on this parameter down below.

4.2.2 Estimation of Extinction Ratio

The nonidealities of the polarizing parameters has an impact on the observed pixel's polarizing efficiency that is quantified by the extinction ratio ER . The error in ER can also be represented by combining Equations 2.4 and 3.9 as a function of the above additive term ξ as follows:

$$ER = \frac{1 + \sqrt{(a_{1,i} + \xi)^2 + (a_{2,i} + \xi)^2}}{1 - \sqrt{(a_{1,i} + \xi)^2 + (a_{2,i} + \xi)^2}}. \quad (4.3)$$

Let us consider the estimated extinction ratio in the θ -oriented channel, in the four cases, Equation 4.3 can be plotted as shown in Figure 4.4.

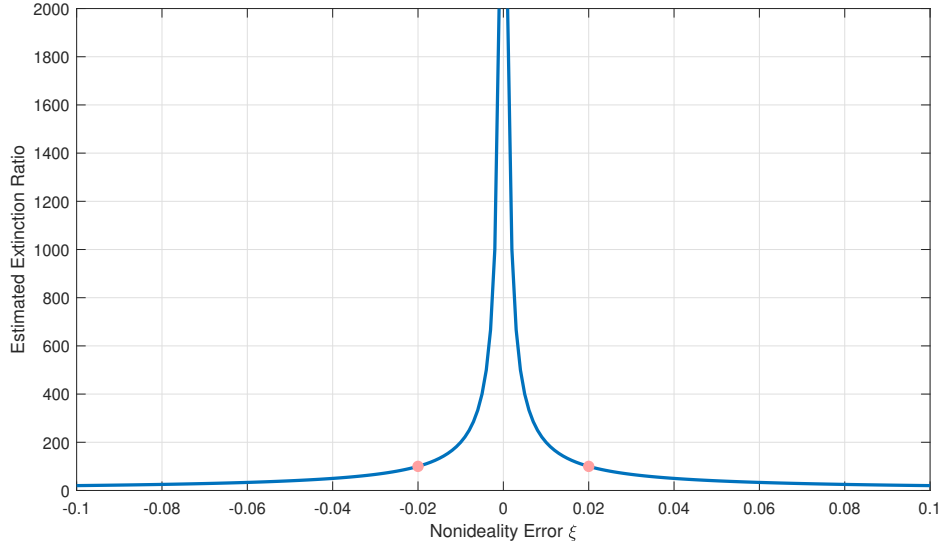


FIGURE 4.4: Estimated Extinction Ratio as a function of a nonideality error. The point when, $\xi = \pm 0.02$ or $ER = 100$, is defined as the turning point between two regions.

In the proposed case study, it can be seen that the estimated extinction ratio ER has a hyperbolic relationship relative to the nonideality error ξ . Let us define the curve's turning point as the value when the extinction ratio allows approaching the condition number to the theoretical minimum [119], that is, $ER = 100$ or $D = 0.98$, instead of the point of intersection of the bisector of each branch. Under this criterion and considering the double asymptotic behavior, which leads to a high nonlinearity, the curve may be divided in two regions:

- High-steepness region: For every $\xi \in [-0.02, 0.02]$ or $ER \geq 100$, the curve exhibits an accelerated growth/decay, i.e., small variations in ξ results in a great variation in the estimated ER , growing to infinity (mathematical singularity or physical polarizing ideality) when the nonideality error equals zero.

- Low-steepness region: For every $\xi \in [-0.1, -0.02] \cup [0.02, 0.1]$ or $ER < 100$, the curve exhibits slower growth/decay, therefore, when the error moves away from zero, this results in smaller variations in the estimated ER .

4.3 Practical evaluation

4.3.1 Characterization Setup

For this study, the optical setup is the same as shown in Figure 3.1, and replicated for two different monochrome cameras, one with onglass and the other with onchip assembly. These assembly procedures are detailed in subsection 2.3.2. The two cameras are:

- Camera A (same as in chapter 3): assembled by 4D Technology, and employing a Sony IMX174 CMOS sensor, coupled with a polarization filter (onchip assembly).
- Camera B: assembled by Lucid Vision Labs, featuring a Sony IMX250MZR sensor [89] (onglass assembly).

The integration time of the cameras is used to modulate the light intensity. The resolution is set to 12 bits for both cameras. A tungsten halogen light source is connected to a Ø6" Newport 819D-SF-6 integrating sphere. It is supposed to remove any polarization signature from the light and produce a spatially uniform, uncollimated light beam. A high extinction ratio linear polarizer is mounted on a rotational stage as the reference. The polarizer's orientation axis is aligned to the mean value of the 0° -oriented channel of the cameras. It avoids globally the orientation bias between the mechanical setup and the camera. Following the recommendations pointed out in Section 3.6, we select M training measurements (each averaged over 100 images), four polarization angles as input, and two different integration times. We collect the data for both cameras A and B, and use Eq. 3.8 to characterize the analysis vector components.

The objective coupled to the cameras is a Navitar model MVL7000 featuring a manual full-stop scale. The diaphragm's aperture area is doubled/halved from one graduation to the next. It allows the f-number to be set continuously throughout six graduated markers from $f/2.5$ to $f/16$. Likewise, the effective focal length (EFL) can also be set continuously inbetween the maximum graduations: $18mm$ to $108mm$, and with a fixed back focal length (BFL) at $19.2mm$. To maximize the spatial uniformity of the data, a region of interest is selected so that 1000×1000 pixels are taken from the center of the sensor area. This region size represents 43% and 20% of the entire sensor resolution in cameras A and B, respectively.

4.3.2 Photometric characterization of PFA cameras

To perform the photometric characterization of the PFA cameras, the camera responses are collected for a uniform and broadband light (using the setup pointed out in Subsection 4.3) but without the reference polarizer. In an ideal case, it would be expected to obtain

distributions with standard deviations approaching zero, as well as an overlapping between the mean value of the four channels.

Figure 4.5 shows the flat-field response in camera A. It can be seen that all the standard deviations are roughly in the same order of magnitude. The 135° -oriented channel exhibits lower photometric values than the others. Figure 4.6 shows the flat-field response in camera B. It can be seen that all the standard deviations are in the same order of magnitude but more than four times lower than in camera A. For this camera, the 90° -oriented channel exhibits higher photometric values than the other channels. The characterization of each channel with co-polarized light yielded consistency among the four means, as well as reduction of the standard deviations. Therefore, it can be verified that the light output of the sphere does not have a particular polarization state.

The deviation of intensity among the polarization channels leads to a checkerboard-like pattern (division-of-focal plane fixed pattern) that is visible in the raw image. Therefore, if this pattern has an effect in the polarimetric parameters after the polarimetric calibration, a photometric calibration must be performed.

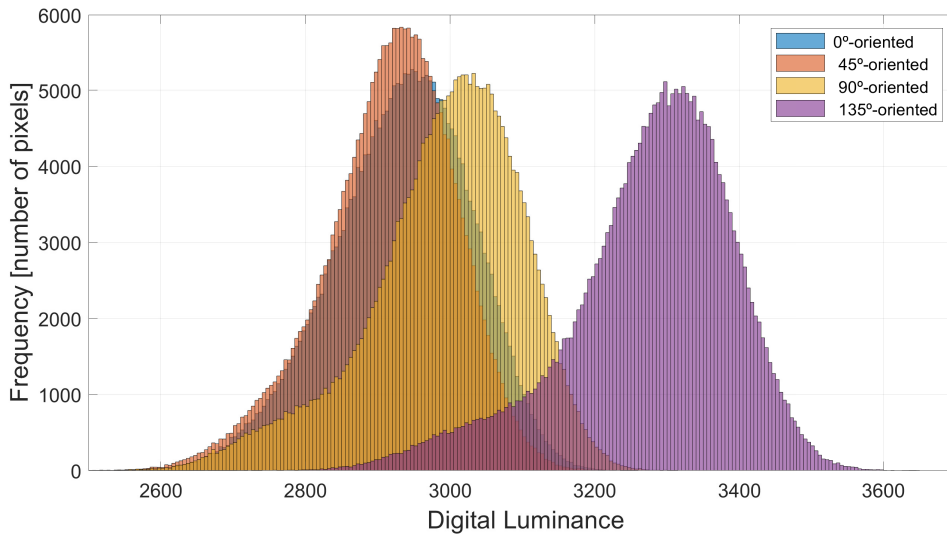


FIGURE 4.5: Histogram of the camera responses by orientation channel in camera A to a uniform illumination. (plotted with 544 bins). The means and the standard deviations $mean[std]$ are: 0° -oriented: 2931 [96.2], 45° -oriented: 2915 [92], 90° -oriented: 2992 [107.6], 135° -oriented: 3275 [117.16].

4.3.3 Polarimetric characterization of PFA cameras

The analyzer matrix $\mathbf{W}$ of each super pixel is determined for each combination of f-number and focal length. We have a total of 36 lens configurations (6 f-numbers and 6 focal lengths), so that the range of both objective's optical parameters is swept. Once each lens configuration is characterized, one can evaluate the figure of merits in terms of analysis vector components. To express the calculated values in terms of unit vectors, one can

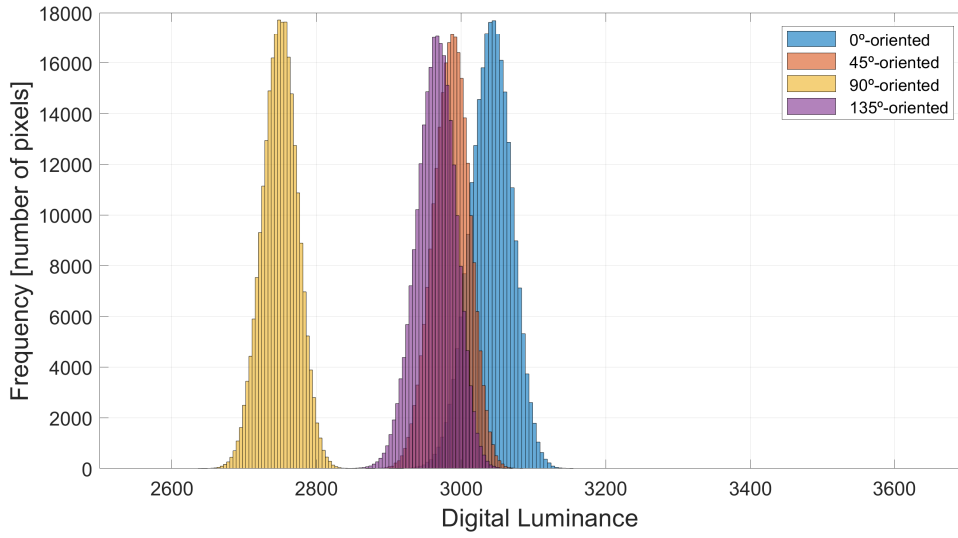


FIGURE 4.6: Histogram of the camera responses by orientation channel in camera B to a uniform illumination. (plotted with 50 bins). The means and the standard deviations $mean[std]$ are: 0° -oriented: 3041 [28.51], 45° -oriented: 2985 [24.92], 90° -oriented: 2749 [24], 135° -oriented: 2964 [26.7].

choose to normalize the raw data with the average value over the area of interest, such that:

$$\hat{\mathbf{I}} = \frac{\mathbf{I}}{\langle \mathbf{I}_{0^\circ} \rangle}, \quad (4.4)$$

where, $\mathbf{I}$ represent the 1000×1000 pixels raw image, the hat ($\hat{\cdot}$) denotes the normalized values, the subscript indicates the measurements with co-polarized light in the 0° -oriented channel, and the symbol $\langle \cdot \rangle$ denotes the average value. Likewise, the Stokes vectors inputs are normalized as indicated in Eq. 1.16. Under this approach, here below there is the calculated mean analyzer matrix $\mathbf{W}_{\text{mean}}$ synthesized by applying Eq. 3.8, along with the corresponding standard deviation $\mathbf{Std}$, for a f-number of $f/5.6$ and a focal length of $40mm$.

$$\mathbf{W}_{\text{A,mean}} = \frac{1}{2} \begin{bmatrix} 1.01 & 0.94 & -0.05 & 0 \\ 1.02 & -0.14 & 0.90 & 0 \\ 1.01 & -0.91 & 0.08 & 0 \\ 1.01 & 0.07 & -0.92 & 0 \end{bmatrix}.$$

such that:

$$\mathbf{Std}_{\mathbf{W}_{\text{A}}} = \begin{bmatrix} 0.04 & 0.04 & 0.01 & 0 \\ 0.03 & 0.02 & 0.01 & 0 \\ 0.03 & 0.04 & 0.02 & 0 \\ 0.03 & 0.02 & 0.04 & 0 \end{bmatrix}.$$

$$\mathbf{W}_{\text{B,mean}} = \frac{1}{2} \begin{bmatrix} 1.00 & 1.00 & -0.01 & 0 \\ 1.01 & 0.00 & 1.01 & 0 \\ 1.01 & -1.00 & 0.00 & 0 \\ 1.01 & 0.00 & -1.01 & 0 \end{bmatrix}.$$

such that:

$$\mathbf{Std}_{\mathbf{W}_B} = \begin{bmatrix} 0.01 & 0.01 & 0.00 & 0 \\ 0.01 & 0.00 & 0.01 & 0 \\ 0.01 & 0.01 & 0.00 & 0 \\ 0.01 & 0.00 & 0.01 & 0 \end{bmatrix}.$$

The analyzer matrices $\mathbf{W}_{\text{mean}}$ can be approximated such that it is characterized in each channel by numerical additive errors, following the relation:

$$\mathbf{W}_{\text{mean}} \approx \frac{1}{2} \begin{bmatrix} 0 & \xi_0 & \xi_0 & 0 \\ 0 & \xi_{45} & \xi_{45} & 0 \\ 0 & \xi_{90} & \xi_{90} & 0 \\ 0 & \xi_{135} & \xi_{135} & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}. \quad (4.5)$$

By using the approximation of Equation 4.5, one can obtain the errors in the four channels in camera A such that: ξ_θ : $\xi_0 \approx -0.05$, $\xi_{45} \approx -0.12$, $\xi_{90} \approx 0.08$, and $\xi_{135} \approx 0.07$. As a result, the nonidealities errors are located in the low-steepness region defined in Figure 4.4. The difference in the numerical errors between the orientation channels corresponds to estimated analyzer angles unevenly distributed over the half circle. Moreover, from the pixel perspective, the estimation of the extinction ratio is unbalanced, which counteracts the assumption of the Stokes vector being the same in the super pixel. In the array $\mathbf{Std}_{\mathbf{W}_A}$ containing the standard deviation of the corresponding vector components, it can be seen that the deviation among the parameters is different by columns, and it is at most 25%.

On the contrary, by approximating the analyzer matrix $\mathbf{W}_{B,\text{mean}}$ under the same above criteria, one gets the numerical additive errors ξ_θ : $\xi_0 \approx -0.01$, $\xi_{45} \approx -0.01$, $\xi_{90} \approx 0.01$, and $\xi_{135} \approx 0.01$, thus, the nonidealities errors are located in the defined high-steepness region. As a result, the estimated extinction ratios will be located in fast growth/decay response as shown in Figure 4.4. In the array $\mathbf{Std}_{\mathbf{W}_B}$, it can be seen that the standard deviations are consistent and lower than in camera A. A similar tendency of deviation is observed in the 36 setups, for both cameras.

Overall evaluation of the analyzer matrix

To represent the error of the twelve entries of the estimated analyzer matrix $\mathbf{W}_{\text{est}}$ in a unified value, the figure of merit E_W is defined as the Frobenius norm of the difference between the ideal and the estimated mean analyzer matrix such that:

$$E_W = \|\mathbf{W}_{\text{est}} - \mathbf{W}_{\text{ideal}}\|_F. \quad (4.6)$$

To depict graphically the evolution of the defined analyzer matrix error E_W as a function of the f-number, Figures 4.7 and 4.8 show the box plot for an EFL of 40mm, for cameras A and B respectively. The central red mark indicates the median. The boxes extend from the 25% quartiles to the 75% quartiles. The whiskers indicate the full ranges of the data excluding the outliers (values away from the mean 2.69 times the standard deviation) that are shown with red crosses. It appears that E_W is nearly constant for

the two cameras when changing from high f-numbers down to $f/4$. For cameras A, the magnitude of errors are roughly ten times higher than camera B. This behavior is observed for all the EFL's for both cameras. In the setup $f/2.5$, camera A exhibits an increase in the number of outliers, whereas in camera B the median is shifted significantly. Moreover, the results obtained in camera B are consistent with those presented by Lane *et al.* [86] who tested the same sensor in the range $f/16$ to $f/4$.

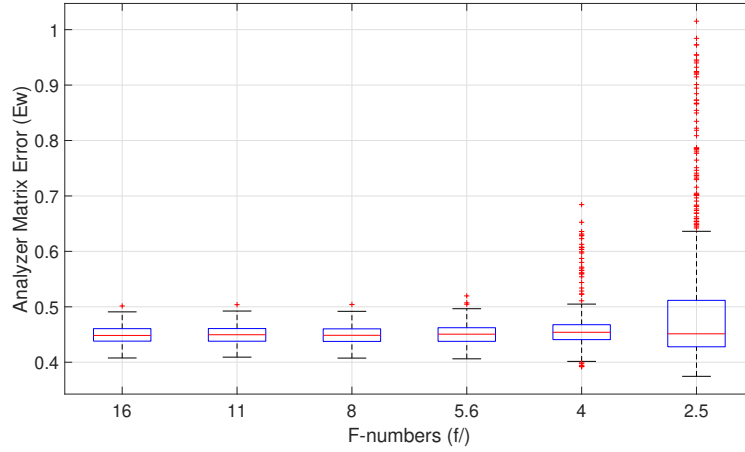


FIGURE 4.7: Analyzer matrix error variation as a function of the f-number for a fixed effective focal length of $40mm$ in Camera A.

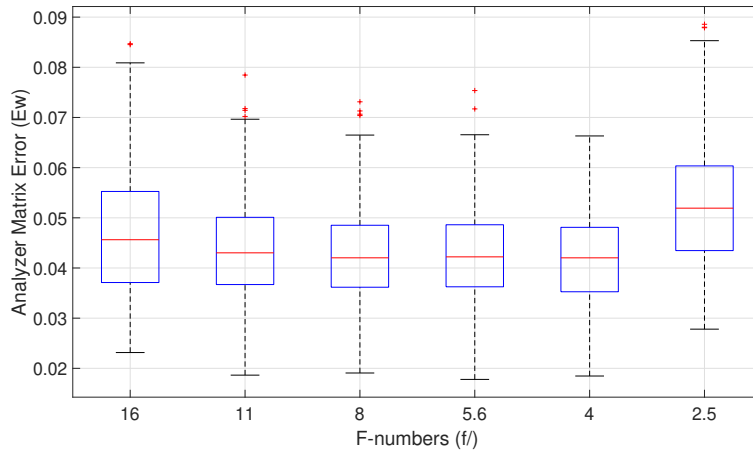


FIGURE 4.8: Analyzer matrix error variation as a function of the f-number for a fixed effective focal length of $40mm$ in Camera B.

Cartographic representation of the extinction ratio

One can also evaluate the polarizing parameters that are obtained as a function of the analyzer vector, $\mathbf{A}$ as indicated in Eq. 3.9 and Eq. 3.10. For example, maps of the extinction ratio ER can be depicted to visually illustrate their spatial distribution, for each channel in the same optical setup.

In Figure 4.9, it is seen that the extinction ratio of camera A has a different spatial pattern for each θ -oriented channel. Due to the cropping constraints, no vignetting effect can be observed. Moreover, the field of view achievable with the set distance between the source and the sensor prevents rays of light from diverging in angles that would make them bounce at the edge of the rotational stage. A similar pattern of nonuniformity is observed in all setups, with the difference that the mean value of ER can be as high as 29. The second column in Table 4.1 contains the average estimated angles for the four orientation channels, showing that the deviation from the nominal values is different among the channels and can be as high as 4° .

Figure 4.10 represents the spatial distribution of the extinction ratio for camera B, excluding outliers by applying the mean method (i.e. excluding values of more than three standard deviations from the mean). These data filtering allows perceiving spatial changes that with much higher values would not be possible. The average extinction ratio exhibits different patterns of spatial inhomogeneity among the channels and with a dynamic range much greater than in camera A, as well as the minimum of the extinction ratios in the four channels. A similar behavior is observed in the 36 setups. The only difference is that the mean value of ER can be as high as $3.6e+10$ and as low as 97. Table 4.1 contains the average estimated angle for the four orientation channels, showing that the deviation from the nominal values is at most 0.1° and the standard deviation is lower than in camera A.

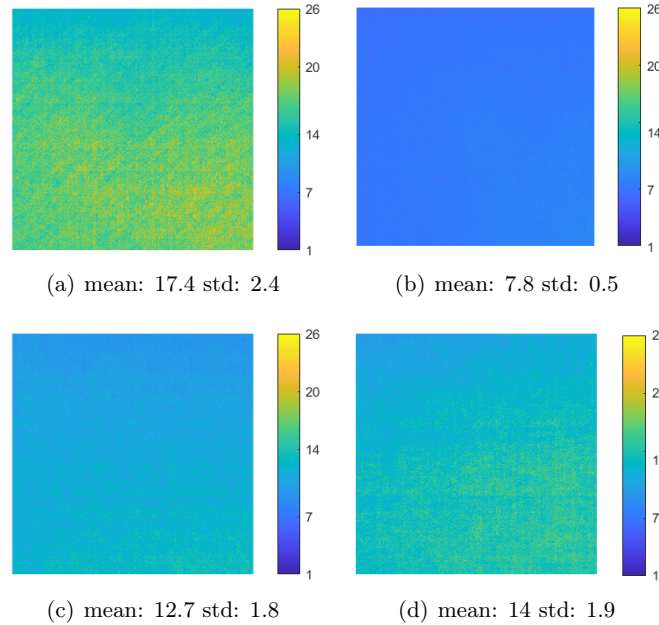


FIGURE 4.9: Maps of the Extinction Ratio ER of the PFA sensors for f-number: $f/5.6$ and focal length: $40mm$ for camera A: a) 0° -oriented, b) 45° -oriented, c) 90° -oriented, d) 135° -oriented.

Evaluation of extinction ratio by orientation channel

In addition to the boxplots representation which combines the error information of the four channels, it is also possible to break down the evolution of the extinction ratio by orientation

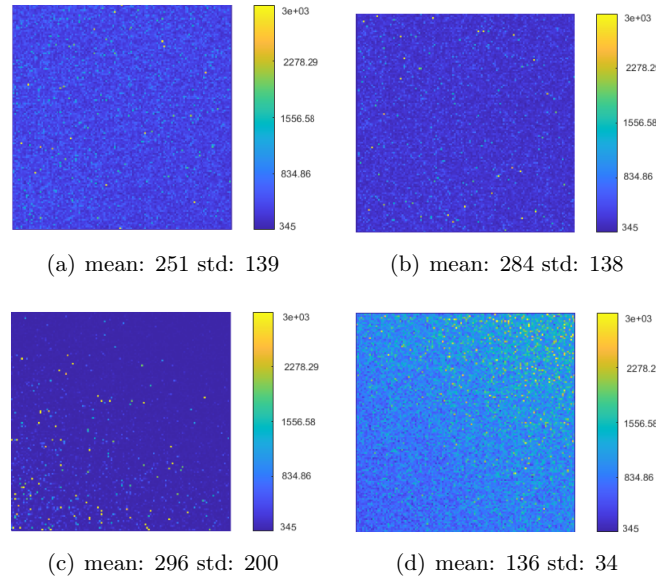


FIGURE 4.10: Maps of the Extinction Ratio ER of the PFA sensors for f-number: $f/5.6$ and focal length: $40mm$ for camera B: a) 0° -oriented, b) 45° -oriented, c) 90° -oriented, d) 135° -oriented. Values are represented after outliers removal.

Orientation Channels	Camera A <i>Mean [Std]</i>	Camera B <i>Mean [Std]</i>
0° -oriented	0.05 [0.34]	-0.04 [0.16]
45° -oriented	48.08 [0.63]	44.80 [0.17]
90° -oriented	85.98 [0.60]	89.86 [0.17]
135° -oriented	137.32 [0.63]	134.11 [0.15]

TABLE 4.1: Average orientations of the PFA sensors in camera A and B for f-number: $f/5.6$ and focal length: $40mm$.

channel as the optical setup is changed. Figure 4.11 shows camera A's ER degradation curves as a function of both f-number and effective focal length. In the varying f-number plot; Subfigure 4.11(a), the distance between the f-numbers has been normalized to the units of area that it is increased from one stop to the other (i.e. twice as much). It can be seen that the four channels have different levels and obey a slight decrease more consistent than in Figure 4.7, especially when transitioning to the setup $f/2.5$.

On the other side, in the varying-EFL plot; Subfigure 4.11(b), it can be seen that the four channels also have different levels of ER but they exhibit more consistency from one transition to another, which could be approximated as constant in the four cases. In other words, when the subject is considered to be infinitely far away, the focal length is directly proportional to the magnification and inversely proportional to the optical power, but have little effect to define a principal angle of incidence, hence, the impact of changing the focal length can be neglected. This criterion is backed-up by Lane *et al.* [86] whose results indicate a lack of focal length dependency.

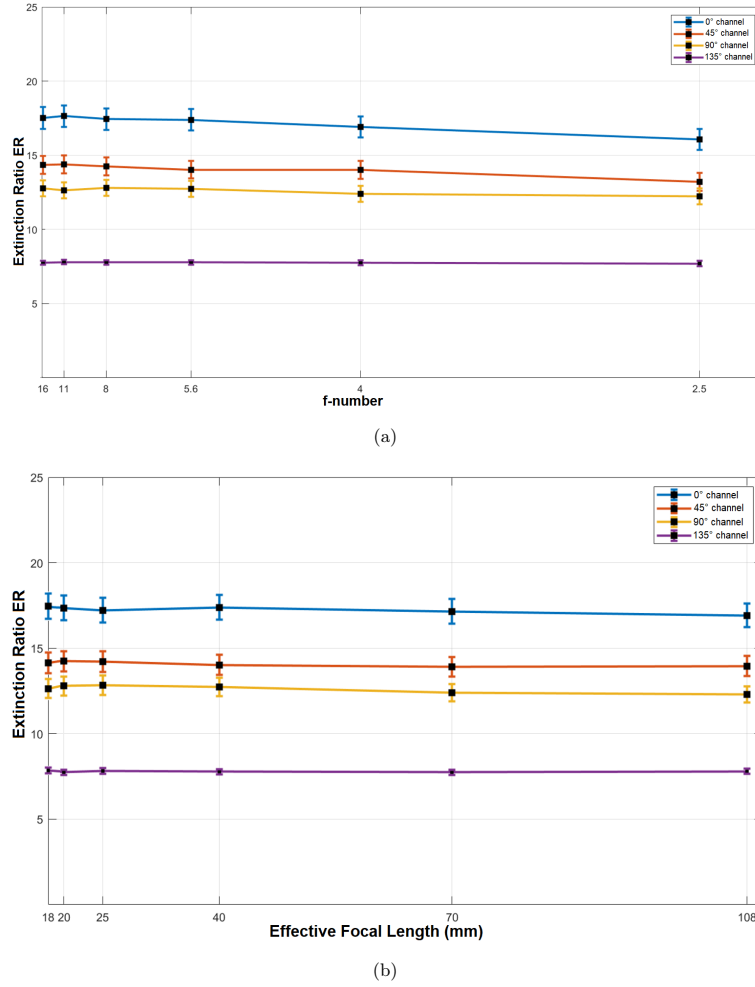


FIGURE 4.11: (a) Extinction ratio versus f-number for a fixed effective focal length of 40mm in camera A. The f-number graduation has been normalized to the units of area that is increased from one stop to another. (b) Extinction ratio versus effective focal length for a fixed f-number of $f/5.6$. Error bars are proportional to the spatial standard deviation.

Figure 4.12 shows camera B's ER degradation curve as a function of both f-number; Subfigure 4.12(a) and focal length; Subfigure 4.12(b), sweeping the whole range in the two scales, after the outliers' removal. Unlike camera A, one notices an irregular behavior in the values of the extinction ratio, which is due to the limitation of the estimators in the number of decimals used to perform the computation approaching a mathematical singularity. In this case it is more convenient to define the PSA's figure of merit in terms of the diattenuation in which case the values are in the interval $D \in [0.97, 0.99]$.

Analysis of the results

Based on the order of magnitude of the nonideality error ξ of Camera A's polarizing figure of merit, it can be said to be a *calibrable* camera. This means that with the proper characterization of the pixel's polarizing properties, the superpixel calibration function will output an image with optimum settings [119] as discussed in Chapter 3.

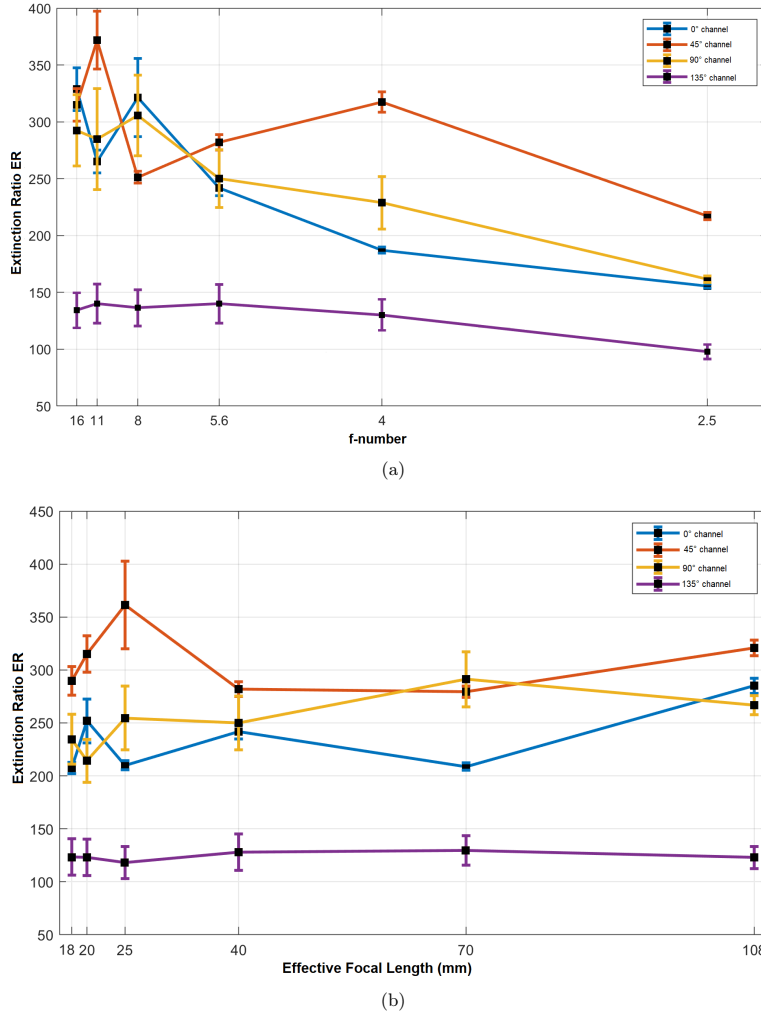


FIGURE 4.12: (a) Extinction ratio versus f-number for a fixed focal length of 40mm in Camera B. The f-number's graduation has been normalized to the units of area that it is increased from one stop to the other. (b) Extinction ratio versus focal length for a fixed f-number of $f/5.6$.

Conversely, Camera B can be said to be a *near-to-ideal*, as it exhibits extinction ratios above 100. As a result, the micropolarizers can be approximated as ideal [119] without a heavy cost in error [73]. In such a case, the calibration function will not have a significant impact on the output, i.e. the corrected values $\mathbf{I}'$ will be the same as the raw intensity values $\mathbf{I}$ after applying the super pixel calibration function.

4.3.4 Analysis of the illumination source setup

The above characterization has been done with respect to a uniform target in a cropped sensor area to irradiate the region of interest homogeneously. Some experimental setups have been proposed in the literature to perform the polarimetric calibration by using nonuniform spatial-distributed light [73, 84] to simplify the procedure by using less expensive sources of light. Nevertheless, any illumination setup, even with the use of an integrating

sphere has sources of residual inhomogeneities and that is more the case for other types of sources such as a LED light bulb masked by an optical diffuser [73].

To evaluate the impact of the light source uniformity in the characterization procedure, the superpixel calibration method is conducted with training data obtained under different levels of nonuniformity, which in turn, are gradually decreased until that obtained with the bare integrating sphere. To define a case study, we emulate the undesired hotspot effect or vignetting, referring to the brightness attenuation away from the image center. This phenomenon typically occurs due to the reflection at lens mount, camera tilt, the zooming range, and/or due to distance between the sensor and the light source [120].

In this subsection the experimental setup is used as described in Subsection 4.3 with Camera A, with the addition of four different optical filters between the integrating sphere's output and the rotating polarizer as shown in Figure 4.13. These mechanical optical filters were made up by printing a vignetting pattern in a translucent adhesive sheet manufactured by AVERY. The vignetting pattern was designed in the software Paint.net, with a diameter equal to that of the reference polarizer, and each of which is characterized by the density (1 is the highest contrast, 0 lowest contrast). The density is reduced 25% from one to another.

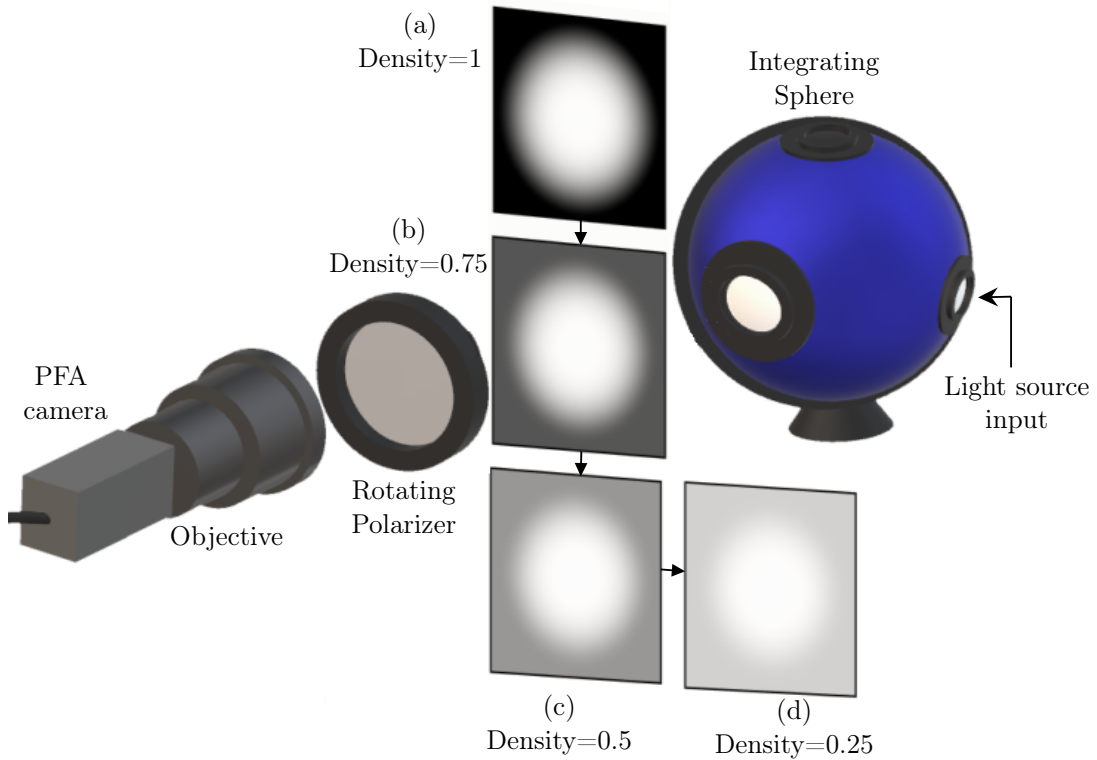


FIGURE 4.13: Acquisition setup used to evaluate the impact of light nonuniformity in the characterization of a PFA camera. Pattern (a): density=1, Pattern (b): density=0,75, Pattern (c): density=0,5, Pattern (d): density=0,25.

Figure 4.14 depicts the said illumination patterns in the zero-oriented channel when

illuminated with fully zero-oriented polarized light. It can be seen that as spatial standard deviation decreases as the vignetting is faded. The other three orientation channels in the same condition show similar patterns and are not shown for brevity.

To fairly compare the above nonuniform conditions to that of the bare integrating sphere, we establish a metric for spatial nonuniformity of illumination ΔE [39]. It is averaged between the four orientation channels when impinged with co-polarized light, such that:

$$\Delta E[\%] = \frac{1}{4} \sum_{i=1}^4 \frac{I_{\theta_i, \max} - I_{\theta_i, \min}}{\mu_{E, \theta_i}} \cdot 100, \quad \theta_i \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ\}. \quad (4.7)$$

Where $I_{\max}$ and $I_{\min}$ represent the maximum and minimum intensity values, respectively, and μ_E the average intensity inside the region of interest in the θ -oriented channel.

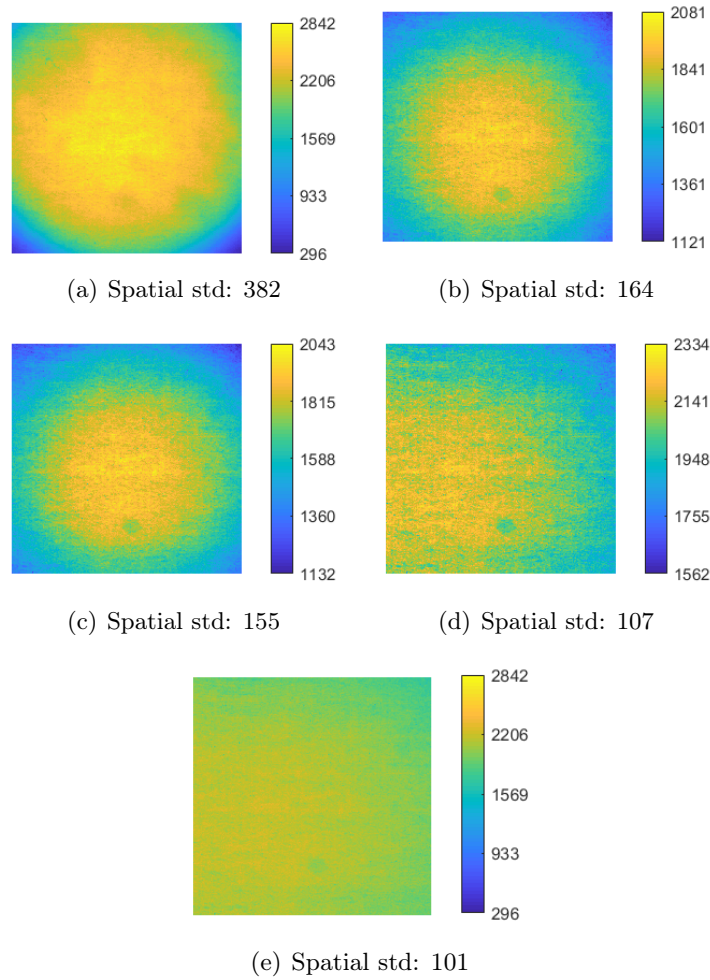


FIGURE 4.14: Nonuniform intensity patterns with hotspot effect in the zero-oriented channel when illuminated with fully zero-oriented polarized light in a region of 600x600 pixels. The color bars are depicted with different levels to let an easy visualization of each pattern. (a) density=1, Pattern (b): density=0,75, Pattern (c): density=0,5, Pattern (d): density=0,25, (d): Bare sphere.

Table 4.2 shows ΔE for the four vignetting illumination conditions in descending order, along with that of the bare integrating sphere; which is the highest achievable uniformity.

Illumination pattern	ΔE [%]
Pattern (a)	114 %
Pattern (b)	56 %
Pattern (c)	53 %
Pattern (d)	37 %
Bare sphere	36 %

TABLE 4.2: Channel-averaged spatial nonuniformity of illumination ΔE for five illumination conditions.

The test image is obtained out of 20° -oriented uniform and fully polarized light capture as a test image in order to evaluate the effect of performing calibration with the parameters characterized by using the five illumination conditions. Figures 4.15 and 4.16 show the Root Mean Square Error in both the Degree and Angle of Polarization as a function of the nonuniformity metric.

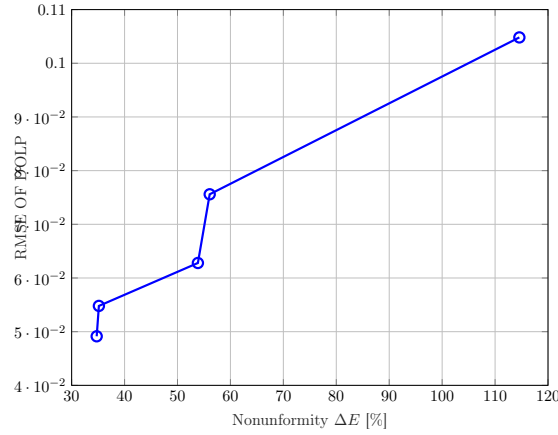


FIGURE 4.15: Root Mean Square Error in the DOLP as a function of the nonuniformity metric ΔE [%].

It can be seen that the error in both polarimetric parameters increases with the nonuniformity ΔE . This response is because the output light scattered by the concave interior of the integrating sphere is evenly distributed in space not only in intensity but also free from the effects of incident direction, and incident position, that is, the integrating sphere exhibits Lambertian reflectance. Figure 4.17 shows a schematic representation of the optical path due to two types of Lambertian sources, in a way that the propagation direction of the incoming rays is uniform-distributed. Red arrows depict a near-field point source that contains rays with incident angles likely to produce crosstalk (dashed arrows) as shown in Figures 4.1 and 4.2 but without being the principal propagation direction. Besides, if the source located far enough from the PFA sensor, one can consider a far-field Lambertian source depicted by the green arrows. In this case, as the radius of the spherical wavefront

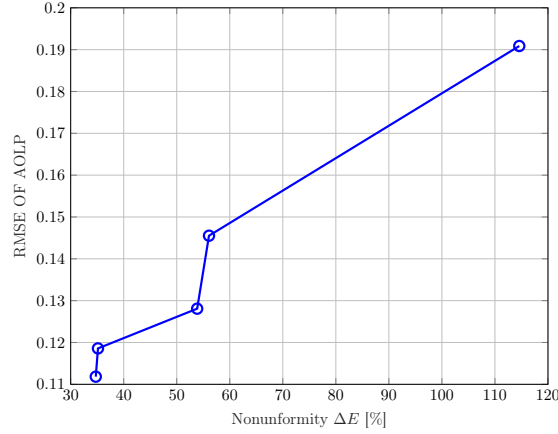


FIGURE 4.16: Root Mean Square Error in the Angle of Linear Polarization as a function of the nonuniformity metric ΔE [%].

of light that strikes the sensor increases, such a wavefront can be considered planar over the sensor and with a normal incidence.

Conversely, any other non-Lambertian emitter surface, e.g., the above vignetting filters, despite having a smooth gradient pattern, are prone to emit an irregular incident propagation patterns and hence introducing a bias in polarimetry due to an uneven optical crosstalk, especially in PFA on-glass sensor. Figure 4.18 shows a schematic representation of a random optical path due to a non-Lambertian source. The three parallel dashed arrows depict a given principal propagation direction, while the one downward-sloping dashed arrow represents an unbalance in the propagation directions toward the opposite side of the sensor. Therefore, it is recommended to maximize the uniformity of the illumination source to the extent possible, as well as establish a trade-off of the precision required if a light source other than an integrating sphere is to be used.

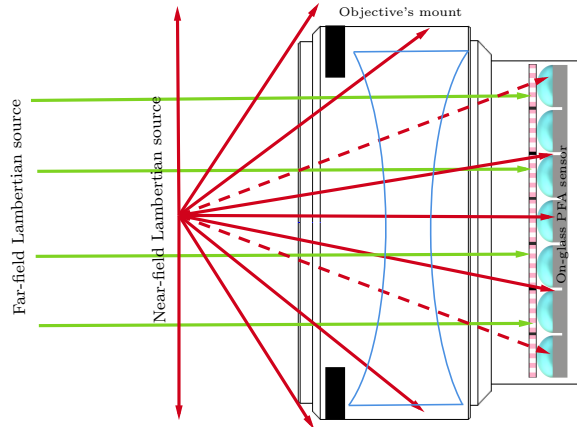


FIGURE 4.17: Schematic representation of the optical path due to two of Lambertian emitters. Red and green arrows depict a near-field and a far-field (green) Lambertian sources respectively. In both cases, the propagation directions are uniform-distributed. Dashed arrows depict rays with higher opportunity to produce crosstalk.

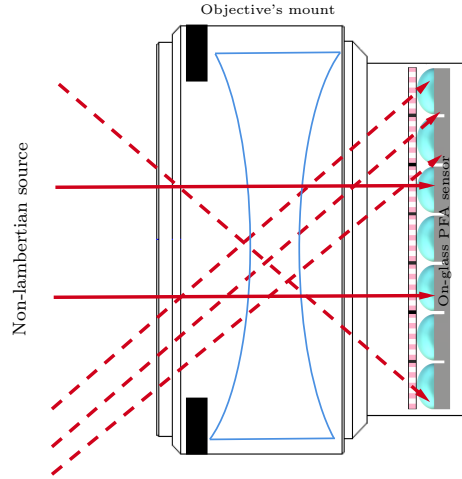


FIGURE 4.18: Schematic representation of a random optical path due to a non-Lambertian source. The three parallel arrows depict a given principal propagation direction, while the others depict any residual directions. Dashed arrows depict rays with higher opportunity to produce crosstalk.

4.4 Proposed Calibration Method

In this section, we propose a method to relief the characterization procedure for a *calibrable* camera when the f-number is varying and the effective focal length is fixed.

Based on the results obtained for Camera A in Section 4.3.3, it is seen that the response of the analyzer vector components are well-behaved among the four orientation channels from one f-stop to the other, one gets an approximate constant response taken as reference Figure 4.7. This fact suggests that it is possible to use a value interpolation between two already characterized f-numbers (out of at least four measurements each) and predict all values in all in-between.

To try the feasibility of the above scheme, we performed a conventional characterization in the middle of the full-stops graduated scale of our objective, in other words, in the missing numbers of the one-half-stop scale namely, $f/3.3$, $f/4.8$, $f/6.7$, $f/9.8$ and $f/13$ for a fixed EFL of $18mm$.

Moreover, a linear interpolation of the pixel's analysis vector $\mathbf{A}$ is performed in a component-to-component fashion from one stop to the other and obtained an interpolated analyzer matrix $\mathbf{W}_{\text{int}}$ for every super pixel, in the same missing numbers of the one-half-stop scale. A 20° -oriented fully polarized light capture is chosen as a test image in order to evaluate the effect of using a conventionally characterized $\mathbf{W}$ versus $\mathbf{W}_{\text{int}}$ in Eq. 3.6 and then in the Stokes vector components as in Eq. 2.6.

Figure 4.19 shows the calculation of the Degree of Linear Polarization (DOLP) and the Angle of Linear Polarization (AOLP) for the uncalibrated, conventionally calibrated and interpolated calibrated case. As expected, the super-pixel method depicted in subfigures (c) and (d) increases the DOLP to the nominal value and reduces the standard deviation as well as the standard deviation on the AOLP. In this sense, the single-axe interpolated calibration method has an error of only 2% respect to the super pixel with a standard deviation slightly higher, which is seen as a subtle nonuniformity in both polarimetric

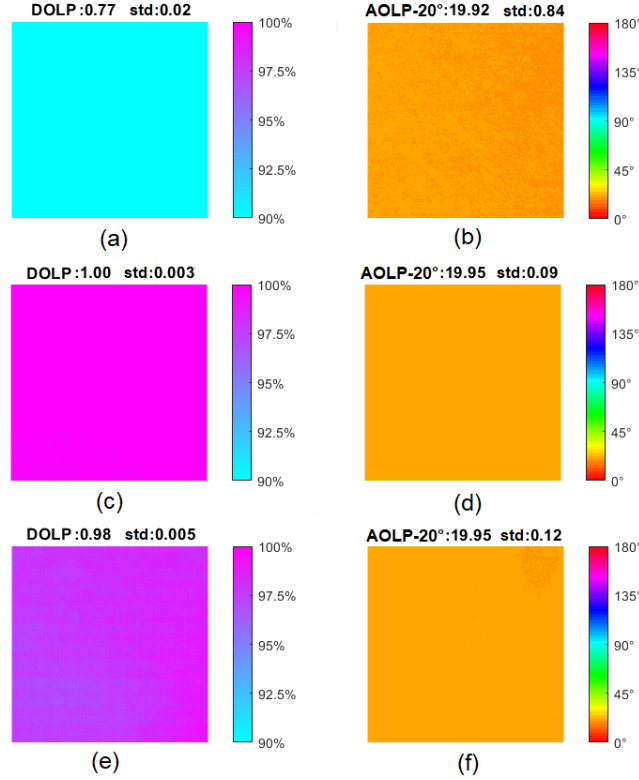


FIGURE 4.19: Degree and Angle of Linear Polarization (left and right columns) calculated for a 20°-oriented fully polarized light capture in a region of 1000x1000 pixels for a focal length of 18mm and f-number of $f/6.7$ in Camera A. (a)-(b) Uncalibrated images. (c)-(d) Conventionally Calibrated images. (e)-(f) Single-axis interpolated calibrated images.

images. For brevity, Table 4.3 contains only the mean values and standard deviation of the DOLP and AOLP for all the f-numbers that were used. It can be seen that the tendency is similar when comparing one method to the other, but with a reduction of the DOLP up to 3%.

4.5 Conclusion

In this chapter, the influence of the optical path due to the use of an objective lens has been studied for two cameras with different ranges of extinction ratio. The analysis of the influence of the illumination source has been conducted, showing that the spatial nonuniformity introduces an error that has to be quantified and considered in the application where the PFA camera is being used. However, it is possible to establish a trade-off between feasibility and cost that may lead to implement characterization setups optimal for a given configuration.

This work also brought the introduction of a new interpolation approach that we have called single interpolated calibration (referring to the f-number) which will allow the user estimating polarimetric images with a precision that is comparable to that provided by conventional characterization methods when using a calibrable camera. This method represents a trade-off that may be useful when several random optical setups or configurations

f/number	Polar <i>metric</i>	Uncalibrated <i>Mean [Std]</i>	Conv. Calibrated <i>Mean [Std]</i>	Inter. Calibrated <i>Mean [Std]</i>
f/3.3	DOLP	0.77 [0.028]	1.00 [0.003]	0.97 [0.004]
	AOLP	20.04° [0.848]	20.01° [0.095]	19.96° [0.133]
f/4.8	DOLP	0.77 [0.026]	1.00 [0.003]	0.97 [0.006]
	AOLP	19.95° [0.844]	19.95° [0.095]	19.95° [0.130]
f/6.7	DOLP	0.77 [0.026]	1.00 [0.003]	0.97 [0.055]
	AOLP	19.92° [0.842]	19.95° [0.096]	19.95° [0.120]
f/9.5	DOLP	0.78 [0.025]	1.00 [0.003]	0.98 [0.005]
	AOLP	19.90° [0.842]	19.91° [0.095]	19.94° [0.113]
f/13	DOLP	0.78 [0.025]	1.00 [0.004]	0.98 [0.004]
	AOLP	19.90° [0.845]	19.94° [0.092]	19.94° [0.108]

TABLE 4.3: Degree and Angle of Linear Polarization calculated for a 20°-oriented fully polarized light capture in a region of 1000x1000 pixels for a focal length of 18mm and along the one-half-stop scale from $f/3.3$ to $f/13$ for: Uncalibrated images, Conventionally calibrated images, and Single-axe interpolated calibrated images.

are to be used, which is the case when using modern objectives with a standard f-stop scale that can be swept as a continuum rather than in steps, i.e. the user may set as many setups as the precision of the rotational actuator allows it (either manually or motorized). In such a case, the number of characterizations in a PFA camera could be overwhelming.

When put to practice, it is essential to have great precision in the objective lenses variation, and it must be motorized rather than manual as in this manuscript. For future work, one might consider the evaluation of a double-axis interpolated calibration so that it is possible to make a prediction of the characterization using any f-number and focal length conjunction.

Conclusions

Summary

Polarimetric Imaging is a non-conventional method that allows to extract additional information from a scene other than color and intensity. Unlike other false color imaging techniques such a thermal imaging where heat can be felt by the sense of touch and can be considered intuitive, the interpretation of polarimetric image requires a prior understanding of the concept of the vector nature of light, as well as the scale that is used to present the polarimetric parameters, which can be separate or combined. This manuscript has been focused in the Division of Focal Plane imaging architecture based in the polarimetric filter array, which has the particularity of having micro-polarizers placed directly on the photosensor array, making it possible to measure a linear Stokes vector in a single acquisition. The main drawback of the DoFP polarimeter technology is that the spatial resolution of the polarimeter images is reduced with respect to the intrinsic sensor resolution in a proportion of four to one. Nevertheless, other architectures have been discussed as in some cases the hybridization allows counteracting some limits and extend the capabilities; for example, to measure circular polarization or increase the image resolution. All these potential configurations make PFA imager a flexible measurement system capable of adapting to several situations.

The PFA imager is perturbed by different sources of inaccuracies such as temporal noise which affects the pixel's response randomly and, the fixed pattern nonuniformities (misleadingly called noise) which is deterministic. The fixed pattern noise uniformities are turned up due to several factors such as the limitation of the nanofabrication process to reproduce consistent features in the micropolarizers, the assembly process, the nature of the incoming light along with the intrinsic radiometric limitations of the underlying photo sensor. All of these sources of perturbation combined lead to errors in the final parameters to be delivered by the imager, what is known as polarimetric degradation. Before performing a polarimetric measurement, the characterization of the analyzer matrix allows quantifying how far from ideal the imager is, and also, it is possible to use this results to correct a target image. The implementation of those two steps are what is being considered in this manuscript as calibration. A PFA imager can be seen as an array of several punctual polarimeters, to this extent, the corresponding analyzer matrices must be characterized and consequently used to correct any measured value.

In this manuscript, two useful terms have been coined to specify the type of calibration that is being performed, namely, the direct Stokes calibration and the calibration from the raw image. The direct Stokes calibration arises from the theory of polarization and

is applicable for all architectures. On the other side, the calibration from the raw image implies the use of a calibration function that allows to correct the image before estimating any polarimetric parameter, which allows the user performing interpolations that otherwise would not be possible. The use of the calibration function known as superpixel has been proven to be the optimum algorithm to carry calibration in PFA cameras, and in turn, it happens to be equivalent to the direct Stokes estimation. Other methods do not improve significantly the performance. Also, the superpixel method allows the user performing polarimetric calibration in a CPFA imager.

In addition, the characterization setup has been also studied to optimize the number of referential acquisitions, which proves very useful, for example, when the camera has to be recalibrated several times or when performing a spectral characterization with a monochromator. The process of characterization has been simplified to avoid redundancy in the measurements, but at the same time to output an analyzer matrix that is suitable for target images of different dynamic ranges. As a result, it has been proposed that images taken at four angles evenly distributed over the half circle replicated at two different levels of illumination yields the best trade-off.

To go further in the simplification of the calibration of a PFA imager, the two assembly architectures have been studied with respect to the optical path due to the objective's f-number and focal length. It has been seen that the new generation of on-chip cameras exhibit polarizing features such that they can be approximated polarimetrically as ideal. Nevertheless, techniques to reduce the temporal noise and the deviations of the response of the orientation channels must be continued to be developed. On the contrary, the former generation of on-glass cameras require calibration which is dependent on the objective's f-number but not on the focal length for the case of uncollimated light at normal incidence. It has also been showed that the analyzer matrix characterization of the on-glass cameras is highly recommended being done with the use of a uniform target such as an integrating sphere since it emulates the properties of a far-field wavefront emitted by a target. It is worthy to mention that the earlier generation of post-process assembled PFA sensors (on-glass) technology is no longer commercially available since the release of new Sony sensors has democratized the access to polarimetric imaging with additional advantages such as much lower price. This thesis was carried during this revolution and is it indented to make the reader understand the transition of the PFA technology.

Perspectives

If polarimetric imaging is going to ever reach our personal devices, it will do under the division of focal plane technology as it is fundamentally compact, embeddable, and it is currently undergoing a significant revolution. It will be probably done under the integration to the CPFA technology, and therefore, such sensor must be able to perform color and polarimetric images without being affected by each other. In this respect, there is also the option of performing the study of spectral-polarimetric imaging and consequently, a thesis is being developed in the laboratory IRIMAS, on this topic. After many years of

development, color imaging still requires calibration and therefore future studies could be focused on the relation of how this sources of inaccuracy might affect the polarimetric performance and how polarimetric features affect color imaging. Also, all the findings must be tested in real-world applications in a way that end users can provide feedback on what the work must be emphasized or oriented.

List of publications and communications

Publications in peer-reviewed journals

1. Yilbert Giménez, Pierre-Jean Lapray, Alban Foulonneau and Laurent Bigué, “Calibration algorithms for polarization filter array camera: survey and evaluation”, *Journal of Electronic Imaging* **29** (4), p. 041011 (13 pages), 2020.
DOI: [10.1117/1.JEI.29.4.041011](https://doi.org/10.1117/1.JEI.29.4.041011).

Oral communications given in international conferences

1. Yilbert Giménez, Pierre-Jean Lapray, Alban Foulonneau and Laurent Bigué, “Calibration for polarization filter array cameras: recent advances”, in *Fourteenth International Conference on Quality Control by Artificial Vision*, C. Cudel, S. Bazeille and N. Verrier Eds., Proc. SPIE **11172**, p. 117216 (6 pages), 2019.
DOI: [10.1117/12.2521752](https://doi.org/10.1117/12.2521752).

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Appendix A

Publication in the Journal of Electronic Imaging

Calibration algorithms for polarization filter array camera: survey and evaluation

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SPIE



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Calibration algorithms for polarization filter array camera: survey and evaluation

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Abstract. A polarization filter array (PFA) camera is an imaging device capable of analyzing the polarization state of light in a snapshot manner. These cameras exhibit spatial variations, i.e., nonuniformity, in their response due to optical imperfections introduced during the nanofabrication process. Calibration is done by computational imaging algorithms to correct the data for radiometric and polarimetric errors. We reviewed existing calibration methods and applied them using a practical optical acquisition setup and a commercially available PFA camera. The goal of the evaluation is first to compare which algorithm performs better with regard to polarization error and then to investigate both the influence of the dynamic range and number of polarization angle stimuli of the training data. To our knowledge, this has not been done in previous work. © The Authors. Published by SPIE under a Creative Commons Attribution 4.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: [10.1117/1.JEI.29.4.041011](https://doi.org/10.1117/1.JEI.29.4.041011)]

Keywords: polarization filter array; division-of-focal-plane polarimeter; characterization; calibration.

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1 Introduction

The electric field that describes electromagnetic radiation can be considered as a vector, whose direction of oscillation is perpendicular to the direction of the wave. This geometrical description of waves is known as polarization.¹ Reptiles or birds, for example, are capable of perceiving polarized information, whereas humans, like any other mammals, are only sensitive to two properties of light: intensity and color. The analysis of the polarization scattered by a scene is known as polarimetric imaging; it yields a complementary performance to classical intensity imaging.² A mathematical tool named Stokes vector is used to describe practically and efficiently the modification of the polarization states when the light travels and interacts with different materials. Polarization analysis using polarimeter instruments becomes increasingly popular in imaging applications, such as classification of materials,³ 3D inspection and reconstruction,⁴ image dehazing,⁵ etc.

In general, polarimeters can be categorized into scanning or snapshot devices, none of which is exempt of tradeoffs and optical imperfections. Scanning polarimeters include the division-of-time (DoT) polarimeter, whereas snapshot polarimeters include the division-of-amplitude, division-of-aperture, and division-of-focal-plane (DoFP).^{6,7} Despite the fact that each approach is arguably suitable for different applications, DoFP is the ideal choice for real-time imaging as it is capable of analyzing light polarization within one sensor integration period, which avoids motion artifacts introduced by DoT polarimeters. These devices lead to a lower cost, simpler design, and higher compactness than other snapshot devices.

The DoFP polarimeters rely on the cell-to-cell coupling of a polarization filter array (PFA) to the imaging system's focal-plane array. PFA technology is a derivative of the filter array imaging principle^{8,9} and was first patented in 1995,¹⁰ but most of the practical implementations and technology advances have been made since 2009. Manufacturing processes are different,¹¹ but commercial sensors (like the SONY IMX250 MZR sensor) tend to have a standardized spatial

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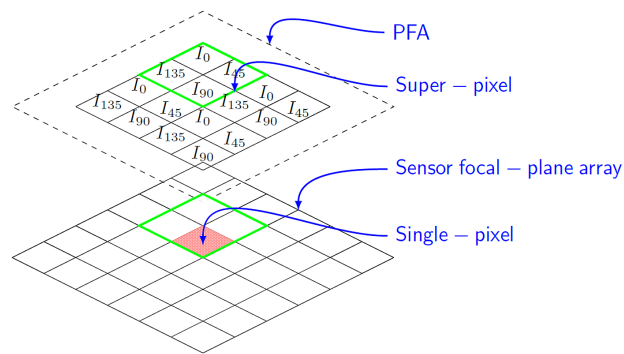


Fig. 1 The camera design considered in this paper is a polarization filter assembly over a monochrome sensor. The sensor matrix is composed of photodiodes and each polarization filter covers one sensor pixel.

arrangement with a repeating pattern of four linear polarizers with orientation axes of 0, 45, 90, and 135 deg¹² (see Fig. 1). About the spatial arrangement, every quadrant is located diagonally to its orthogonal counterpart, but other arrangements exist.¹³ This filter array imaging suffers from sparsity, i.e., each pixel senses only one polarization channel. It introduces instantaneous field-of-view errors¹⁴ when reconstructing 2D polarization scene information from sparse data. Some evolved interpolation methods, like what was done before for the color and spectral domains, have emerged to compensate for these drawbacks.¹¹ The acquisition of intensities through the four linear polarizers makes it possible to estimate the first three Stokes vector components of the input light. To sense circular polarization, i.e., the fourth Stokes component, additional optical elements must be combined with the PFA. Practical implementation of full-Stokes PFA instruments is at the very early stages. In this paper, we will only consider the linear analysis of polarization.

Intrinsically, a silicon sensor (CMOS or CCD) has several sources of errors. Dark current, readout, or salt-and-pepper noises are typical examples. They are corrected using a particular noise pattern, where pixels are susceptible to giving brighter intensities than others when illuminated with homogeneous light. PFA cameras exhibit the noises inherent to silicon sensors, but additional noises due to manufacturing optical imperfections of filters are introduced. Thus, each polarizer has its own optical characteristics, i.e., transmission, diattenuation, and polarization analysis orientation.¹⁵ It results in a pattern noise that could lead to a spatial variation of digital values up to 20% over the whole sensing area.¹⁶ Spatial calibration procedure is thus necessary to compensate for these nonuniformities.^{17,18} We believe that this step is crucial for numerous applications. The uncalibrated camera values could lead to false contrasts or large errors when computer vision algorithms are applied, e.g., material inspection, shape from polarization, index of refraction retrieval, or illuminant direction estimation.

In this paper, we will introduce the global PFA imaging model in Sec. 2, before reviewing calibration procedures in Sec. 3. After defining an experimental setup, we characterize and apply several existing calibration methods on raw data in Sec. 4. Then, we investigate more deeply a calibration procedure by evaluating independently two acquisition criteria in Sec. 5. Conclusion is provided in Sec. 6.

2 PFA Imaging Model

In this section, we enunciate the polarization measurement model coming with PFA cameras. Stokes formalism includes a suitable depiction of polarization states of the light. Four components, arranged in a vector, fully describe the polarization states of light. Since a PFA polarimeter is intrinsically a linear polarization analyzer composed by linear diattenuators, the input Stokes vector of light $\mathbf{S}_{in}$ that reaches the camera can be simplified as $\mathbf{S}_{in} = [S_0 \ S_1 \ S_2 \ 0]^T$. Mueller matrix $\mathbf{M}$, which is a 4×4 matrix, describes the alteration of polarization characteristics of a material element, it can be seen as a transfer function of Stokes vectors. A Stokes vector is linearly transformed, such as $\mathbf{S}_{out} = \mathbf{M}\mathbf{S}_{in}$.

A photodetector array transduces luminous intensity into a camera response I_i , where i is the spatial index over pixels. Thus, only the top row of the Mueller matrix $\mathbf{M}$ is useful to know, which is also called the pixel's analysis vector $\mathbf{A}_i = [a_{0,i} \ a_{1,i} \ a_{2,i} \ 0]$. Thus, if all the first rows of the corresponding Mueller matrices in the sensing area are known, the errors due to nonideal characteristics can be mitigated through calibration. In other words, to carry out precise measurements, the filters do not need to be ideal.

We could now define the imaging model that transforms the incoming Stokes vector into a per-pixel sensed value as follows:

$$I_i = \mathbf{A}_i \mathbf{S}_{\text{in}}. \quad (1)$$

A PFA camera has four different polarization angles of analysis, arranged in a 2×2 pattern. We could then assume a unique spatial position for the four adjacent polarizers, and group the four analysis vectors into an array of vectors (the measurement matrix) that we call $\mathbf{W}_j$, where j indexes the superpixel spatial position. Equation (1) could be extended to this specific configuration:

$$\mathbf{I}_j = \begin{bmatrix} I_{j,0} \\ I_{j,45} \\ I_{j,90} \\ I_{j,135} \end{bmatrix} = \mathbf{W}_j \mathbf{S}_{\text{in}} = \begin{bmatrix} \mathbf{A}_{j,0^\circ} \\ \mathbf{A}_{j,45^\circ} \\ \mathbf{A}_{j,90^\circ} \\ \mathbf{A}_{j,135^\circ} \end{bmatrix} \mathbf{S}_{\text{in}}. \quad (2)$$

Considering Eq. (2), the input Stokes vector can be estimated from intensities using the pseudo-inverse of $\mathbf{W}_j$, assuming that $\mathbf{W}_j$ is known. In most cases, $\mathbf{W}_j$ matrices are not given by PFA manufacturers and should be estimated during a calibration step. We will see in the next section how this model is investigated or even extended in the literature to perform calibration of PFA camera.

3 Calibration Techniques

3.1 Assumptions and General Procedure

Most of the techniques presented in this paper have several important assumptions about the sensor, the calibration optical setup, and the statistical behavior of the signal. One assumes that:

- The sensor operates in the linear regime. Most of the calibration techniques also assume that there is no deviation in the cross-talk effect when coupling the PFA with the focal-plane array, due to either its position relatively to the micro-lens array (below or above), or regarding the sensor orientation with the incident light. Moreover, it supposes that the lens configuration is unique and valid only for a given calibration procedure, i.e., f -number and focal length. Any changes in these parameters afterward will deteriorate the calibration result.
- There is no spectral dependency on $\mathbf{W}$, i.e., retardance is flat over the range of wavelengths considered, and the reference polarizer used to generate training Stokes vectors is perfect (no diattenuation, retardance, or transmission problems¹⁹).
- The acquisition during calibration is not corrupted by temporal noise (a mix of Poisson and Gaussian noises) and that there is no need to apply flat-field procedure to correct training data for residual illuminant spatial deviation.

All the calibration techniques for PFA that we will present share the same global procedure:

1. Inputting a set of light stimuli (training data) with known Stokes vectors to the polarimeter;
2. For each input Stokes stimuli, capture a series of images and average/add them;
3. Estimate the $\mathbf{A}$ vectors or $\mathbf{W}$ matrices, i.e., the polarization properties at each pixel or superpixel, by solving an inverse problem;
4. Compute the gains and offsets from step 3, and apply them to correct raw values.

3.2 Single-Pixel Calibration

Single-pixel calibration is the way to calibrate each pixel independently, without considering the polarization properties of its neighborhood. Powell and Gruev¹⁶ added the offset noise d_i in the model in Eq. (1), to take into account for additive noise during calibration:

$$I_i = \mathbf{A}_i \mathbf{S}_{in} + d_i. \quad (3)$$

Then, a calibration function is applied to pixels as follows:

$$I'_i = g_i(I_i - d_i), \quad (4)$$

where I'_i is the corrected value and $g_i = \frac{\|\mathbf{A}_{ideal}\|}{\|\mathbf{A}_i\|}$ is the normalized gain, where $\mathbf{A}_{ideal}$ and $\mathbf{A}_i$ vectors are assumed to be colinear. As stated in Powell and Gruev,¹⁶ this assumption implies that only transmission errors can be compensated, the single-pixel calibration does not correct for diattenuation and orientation variations (rotational offset) across the PFA structure. Thus, the single-pixel method yields errors when calculating the angle and degree of polarization.

3.3 Superpixel Calibration

Superpixel calibration is a more evolved technique. It uses a relative neighborhood of four pixels to form the superpixel used in the calibration framework; it compensates for deviations in transmission, diattenuation, and orientation. Angle of linear polarization (AOLP) and degree of linear polarization (DOLP) are much more precise with this technique since an in-common pixel correction is done instead of the individual.

Myhre et al.²⁰ use Eq. (2) to calibrate their full-Stokes polarimeter. They assume no additive noise in their model. On the contrary, Powell and Gruev¹⁶ include additive noise in their superpixel calibration procedure:

$$\mathbf{I}_j = \mathbf{W}_j \mathbf{S}_{in} + \begin{bmatrix} \mathbf{d}_{j,0 \text{ deg}} \\ \mathbf{d}_{j,45 \text{ deg}} \\ \mathbf{d}_{j,90 \text{ deg}} \\ \mathbf{d}_{j,135 \text{ deg}} \end{bmatrix}. \quad (5)$$

As for the single pixel, a calibration function is applied as follows:

$$\mathbf{I}'_j = \mathbf{g}_j(\mathbf{I}_j - \mathbf{d}_j), \quad (6)$$

where $\mathbf{g}_j = \mathbf{W}_{ideal} \mathbf{W}_j^+$ gathers the four normalized gains recovered by pseudoinverse operation over $\mathbf{W}_j$.

They found that the superpixel calibration method reduces reconstruction error, in terms of RMSE for DOLP and AOLP, by a factor of around 10 compared to the single-pixel calibration method along with correction of diattenuation and orientation.

3.4 Adjacent Superpixel Calibration

Chen et al.²¹ performed a complementary strategy to the superpixel calibration by adding a computational step at the end the superpixel algorithm. Once Eq. (6) is applied, every single pixel is recalibrated as a function of the weighted average of the four overlapped superpixels neighborhood at the pixel position:

$$I''_j = 1/4[I'_j + I'_{jleft} + I'_{jdiag} + I'_{jabove}], \quad (7)$$

where the terms of average are the calibrated values of the superpixels located to the left, above, and diagonal (adjacent to the latter two), respectively. Authors do not give explanation or justification about doing this average. They argue that the error between the acquired DOLP and the ideal value is reduced in range by a factor of 10. In the same way, the error for the

AOLP is reduced in range by a factor of 4. They also show visually more uniformity in their reconstructed DOLP and AOLP parameters. When applied to a real image, they show that the calibration improves polarization feature contrasts in both intensity and DOLP images. No comparison with the state of the art is done.

3.5 Average Analysis Matrix Calibration

Zhang et al.²² implement the same sequence as Chen et al.²¹ with the same neighborhood, i.e., by calculating the analysis matrix and the offsets from training data. But instead of using an ideal analysis matrix to produce the multiplicative adjustment of the calibration function, an all-encompassing PFA average matrix is used as the common factor in Eq. (6) such that $\mathbf{g}_j = \mathbf{W}_{\text{mean}} \mathbf{W}_j^+$. No analytic criteria are stated for using the mean matrix but this value is closed to the ideal. They apply the calibration to a real image and show qualitative results; the edges are smoother compared to the superpixel method. Quantitatively, they found a thousandth order of magnitude reduction of the RMS error between the mean DOLP and the pixel's DOLP. A quantitative comparison with the superpixel calibration¹⁶ is done using real images, but the authors do not specify how the mean DOLP of an object under study is sampled out of the image.

4 Characterization and Method Comparison

4.1 Acquisition Setup

We use similar optical setup as in Ref. 16 to recover the training and test data. The adjustable shutter is substituted by the camera's integration time to modulate the input signal. The light source is a tungsten-halogen lamp and is provided by an Intralux 4000 module. The light passes through a Thorlabs IS200 Ø2" integrating sphere to generate nominally an assumed uniformed and unpolarized light. A 10LP-VIS-B linear polarizer from Newport is considered as the reference polarizer, and it is rotated by a rotational stage to generate the reference input Stokes vectors. The studied PFA camera is assembled by 4D Technology and employs a Sony IMX174 CMOS sensor coupled with a PFA manufactured by the Moxtek company. Photographic objective's focal length equals 12.5 mm and the f -number was set at $f/1.4$.

With this setup, a group of acquisitions is done:

- Six different intensities, namely, 100%, 50%, 25%, 10%, 5% and 2% (in a way that the 100% maximum intensity equals 75% of the saturation level of the camera) are considered;
- At each intensity, 36 input polarization angles ranging from 0 deg to 175 deg with a step of 5 deg are generated;
- Each couple of intensity/angle acquisition are averaged over 100 images to generate the final data.

In order to maximize the uniformity of the images, a region of interest is selected so that 300×300 pixels are considered at the center of the sensor area.

We make our acquisition database available for further research as supplementary material.

4.2 PFA Characterization

We first characterize the PFA by doing an estimation of the analysis vector parameters $\mathbf{A}_i = [a_{0,i} \ a_{1,i} \ a_{2,i} \ 0]$ and the offset noise d_i , at each pixel i . A least-squares solver involving $N = 54$ instances captured from the acquisition setup described previously (nine equally spaced input polarization angles along with six intensities) is performed for one pixel using

$$(\mathbf{A}_i \ d_i) = (I_{i,1} \ \dots \ I_{i,N}) \begin{pmatrix} \mathbf{S}_{\text{in},1} & \dots & \mathbf{S}_{\text{in},N} \\ 1 & \dots & 1 \end{pmatrix}^+, \quad (8)$$

where “+” means the pseudoinverse, and $\mathbf{S}_{\text{in}}$ is the generated reference input Stokes vector, assumed to be uniform across the region of interest.

Table 1 Mean values and standard deviation (square brackets) of the estimated polarimetric properties: orientation angle α , diattenuation D , and extinction ratio X . These values are estimated over a region of interest of 300×300 pixels at the center of the sensor area.

Polarization	Uncalibrated
Parameter	Mean [Std]
α_0 (deg)	-1.06 [0.53]
α_{45} (deg)	48.60 [0.85]
α_{90} (deg)	88.38 [0.60]
α_{135} (deg)	137.49 [0.65]
D_0	0.839 [0.015]
D_{45}	0.756 [0.016]
D_{90}	0.816 [0.016]
D_{135}	0.815 [0.015]
X_0	11.51 [1.172]
X_{45}	7.23 [0.526]
X_{90}	9.93 [0.955]
X_{135}	9.91 [0.962]

Optical polarization properties, namely the filter's orientation angle α , the diattenuation D , and the extinction ratio X , are then derived from the estimated analysis vector $\mathbf{A}_i$ using Eq. (9):

$$\alpha = 0.5 \arctan\left(\frac{a_{2,i}}{a_{1,i}}\right); \quad D = \frac{a_{1,i}}{[a_{0,i} \cos(2\alpha)]}; \quad X = \frac{1+D}{1-D}. \quad (9)$$

The polarization parameters nonidealities can be quantitatively depicted by their mean value along with their statistical dispersion in the corresponding orientation axes (0 deg, 45 deg, 90 deg, and 135 deg). This makes the assessment possible to see how far they are from their ideal values.

Table 1 summarizes the mean values and the standard deviation across the characterized sensor area. Results show that most of the polarization orientation axis fluctuate and are displaced from their theoretical values. Diattenuation/extinction ratio also exhibit relatively high spatial variations that could lead to imprecision in the polarization measurement if no correction is applied to raw data.

4.3 Calibration Method Comparison

We have implemented the four calibration methods presented in Sec. 3. The test images are acquired when impinging the camera with 20 deg oriented fully linearly polarized light at 100%, 50%, and 5% intensity. One can characterize the performance over the dynamic range (DR) by means of two metrics, namely the root-mean-square error (RMSE) and the peak signal-to-noise ratio (PSNR). Results are presented in Table 2.

The results show that the single-pixel calibration method¹⁶ reduces the error in the S_0 component in an average of 22% but fails to correct S_1 and S_2 , which is in accordance with the inability of the model to correct orientation errors α , as stated by the authors. Conversely, the superpixel calibration method¹⁶ reduces the RMSE in an average of 95% over the three Stokes components. Adjacent superpixel method²¹ yields a 10,000th order of magnitude reduction of the RMSE with respect to the superpixel calibration method. However, the adjacent superpixel

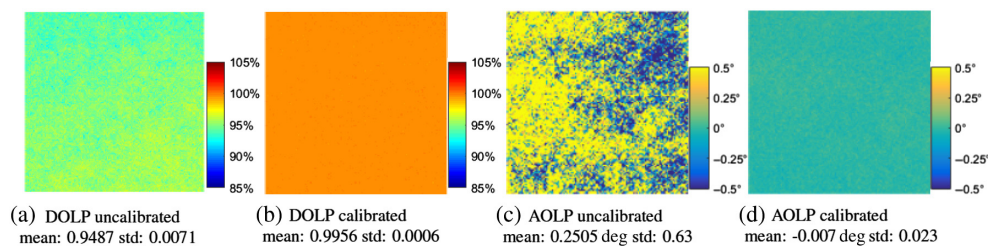
Table 2 Mean RMSE/mean PSNR in terms of the Stokes vector components S_0 , S_1 , and S_2 for the uncalibrated and calibrated responses of a 20 deg oriented fully polarized light capture in a region of 300×300 pixels.

	Uncalibrated	Single-pixel ¹⁶	Superpixel ¹⁶	Adjacent superpixel ²¹	Average matrix ²²
100% Light intensity					
S_0	0.1203/18.39	0.094/20.53	0.0005/65.68	0.0004/67.93	0.1183/18.53
S_1	0.1147/18.80	0.113/18.94	0.0025/52.02	0.0024/52.25	0.1102/19.15
S_2	0.1132/18.92	0.1411/17.01	0.001/61.23	0.001/63.88	0.1082/19.31
50% Light intensity					
S_0	0.1277/17.88	0.1011/19.91	0.0064/43.83	0.0064/43.86	0.1257/18.07
S_1	0.1057/19.52	0.1036/19.70	0.0088/41.07	0.0088/41.13	0.1006/19.95
S_2	0.1053/19.56	0.133/17.52	0.0102/39.81	0.0102/39.86	0.0996/20.03
5% Light intensity					
S_0	0.1231/18.20	0.0965/20.31	0.0033/49.54	0.0026/51.67	0.1209/18.35
S_1	0.1093/19.23	0.1073/19.391	0.006/44.401	0.0048/46.39	0.1043/19.63
S_2	0.1118/19.03	0.1395/17.111	0.0048/46.33	0.0031/50.08	0.1064/19.45

method implies an interpolation across the superpixel neighborhood. Here, the tested image is uniform, so the interpolation step slightly improves the results acting like a smoothing filter. With a real image containing edges in the polarimetric information, the quality of the correction should decrease. For average analysis matrix method,²² it can be seen a reduction of the RMSE in an average of only 4% over the three Stokes components. This might be due to the replacement of an ideal analysis matrix by an uncalibrated-mean analysis matrix, which could carry the errors in the parameters.

Superpixel calibration method is a well-established method. The basis concept of superpixel is a starting point for the other methods. Derived methods do not necessarily have significantly better results.

To illustrate the superpixel method performance, Fig. 2 shows pseudocolor images representing the DOLP and AOLP before and after the superpixel calibration applied to the test image with input polarization light at 0 deg and 100% intensity. When the image is uncalibrated, the DOLP has a mean value significantly lower than 1. The color scale allows to intuitively visualize the heterogeneity of the values in the flat field. The mean AOLP is improved by a shifting near the ideal value and its standard deviation is improved by a factor of 25.

**Fig. 2** Zero-oriented fully polarized light capture and parameter estimations. The region of interest is an area of 300×300 pixels in the center of the sensor.

5 Calibration Setup Optimization

Several previous works investigated different criteria impacting the polarization measurement during the characterization or calibration, such as the f -number and focal length influence,²⁰ the sensor orientation regarding the incident light,²³ or the effect of temporal noise.¹⁹

We believe that optimizing the number of acquisitions during calibration is crucial as the camera has to be recalibrated several times as the f -number or the focal length is changed over time. It appears that a wide variety of calibration methods employ calibration setups that seem to be oversized or not well optimized relatively to these two parameters. This investigation could be useful to make calibration setup lighter in terms of the number of acquisitions.

Here, we propose to investigate the effect of varying the training data configuration over the calibration pipeline. We individually test two criteria, which have not been investigated previously to our knowledge: the DR of input data and the amount of polarization angles used to train the calibration algorithm.

We apply our evaluation on the superpixel calibration from Powell and Gruev¹⁶ as it is the most generic and in-common starting method employed in all the previously reviewed methods.

5.1 Impact of Data Dynamic Range

It is stated in the state of the art that analyzing polarization in a scene with a camera could exhibit pixel regions that sense low irradiance or high polarization degree.²⁴ Thus, one can envisage using several images with different DRs of intensities during characterization and calibration.

To see how DR of training images impacts the calibration result, we propose here to characterize and train the model using six scenarios regarding the DR magnitude: all intensity images combined, only 100%, combination of 100%|50%, 100%|50%|25%, 100%|50%|25%|10%, and 100%|50%|25%|10%|5% intensity levels. We use nine equally spaced input polarization angles for each scenario in order not to be affected by the angle selection criteria, which will be tested in the next subsection in this evaluation step. Test of the calibration is done using only one image at one specific angle (10 deg), which is not in the training sets, and with six different intensities.

Table 3 examines the PSNR reconstruction of the Stokes vector components after applying the calibration over the scenarios. It can be seen that the error has different trends over the three components so it is not obvious that using a high DR increases the performance of the calibration function. Different analyses could be done by looking at the results:

- The three Stokes parameters are not affected similarly by the DR enhancement of training data, the S_0 component is the most affected.
- Using a great variety of DR increases the global performance by looking at the mean values over the tested images. By evaluating an equally weighted average over the three components, the 2-DR image scenario gives the highest PSNR, whereas the standard deviation is inversely proportional to the number of realizations. Intermediate DR images in the set insignificantly increase the calibration performance.
- If dealing with polarization signatures that are mostly close to the noise level of the sensor, it is preferable to include low DR in the training set.

To summarize, adding more than just one DR image to the training setup will enhance globally and significantly the result of calibration which seems intuitive. Using only two different DR seems to be enough to increase significantly the results compared to training with only 100% intensity images, adding more is not judicious.

5.2 Impact of Training Angle Selection

To evaluate the importance of the number of input polarization angle acquisitions made for the characterization, we propose to apply the superpixel calibration over a set of images captured when the camera is illuminated with uniform polarized light at 100% of intensity and by varying both the number of polarization angles for training and the polarization angle for test. To this end, we first train and characterize the analysis vectors using six scenarios: 36, 18, 9, 5, 4, and 3 equally distributed angle images at six different intensities. The latter setup involved the solved

Table 3 Results of the PSNR calculated over the S_0 , S_1 , and S_2 parameters for each scenario. The DR effect of training (row) and testing (column) data is evaluated through these tables. A single angle of 10 deg is used as a test image. Best values are highlighted in bold and worst values in italics.

(a) S_0								
	100%	50%	25%	10%	5%	2%	Mean	[Std]
100%	54.75	40.76	51.46	46.62	45.20	28.08	<i>44.48</i>	[9.40]
100% 50%	60.73	46.74	57.57	52.62	51.20	34.05	50.48	[9.42]
100% 50% 25%	57.02	47.95	58.61	51.31	50.81	34.34	50.01	[8.66]
100% 50% 25% 10%	55.08	48.82	58.23	50.45	50.39	34.52	49.58	[8.17]
100% 50% 25% 10% 5%	54.37	49.21	57.85	50.10	50.20	34.60	49.39	[7.95]
All DR	50.43	52.55	53.78	47.73	48.45	35.14	48.01	[6.71]
(b) S_1								
	100%	50%	25%	10%	5%	2%	Mean	[Std]
100%	55.36	41.46	44.11	48.51	46.48	33.53	<i>44.91</i>	[7.30]
100% 50%	48.63	44.00	47.56	51.20	47.87	34.41	45.61	[5.96]
100% 50% 25%	48.05	44.38	48.10	51.33	47.91	34.52	45.71	[5.91]
100% 50% 25% 10%	48.03	44.39	48.13	51.36	47.93	34.53	45.73	[5.91]
100% 50% 25% 10% 5%	48.01	44.40	48.15	51.37	47.93	34.53	45.73	[5.91]
All DR	46.79	45.35	49.50	51.28	47.86	34.80	45.93	[5.83]
(c) S_2								
	100%	50%	25%	10%	5%	2%	Mean	[Std]
100%	55.90	47.71	52.42	50.79	47.13	39.16	<i>48.85</i>	[5.73]
100% 50%	59.49	49.36	54.22	49.95	46.56	39.60	49.86	[6.76]
100% 50% 25%	60.48	49.93	54.69	49.66	46.37	39.75	50.15	[7.07]
100% 50% 25% 10%	60.79	50.11	54.85	49.59	46.32	39.79	50.24	[7.18]
100% 50% 25% 10% 5%	60.91	50.17	54.92	49.57	46.31	39.80	50.28	[7.21]
All DR	61.63	51.90	55.55	48.69	45.74	40.15	50.61	[7.54]

Eq. (8), which is under-determined as the number of instances is not equal to the number of unknowns. Thus, the said scenario is possible assuming that the dark offsets are small compared to the DR of the camera (12-bit intensity). Dark offset could also be determined experimentally.²⁵

Once the scenarios are trained, we apply the correction on several single-test images corresponding to 36 different input polarization angles (from 0 deg to 175 deg with a step of 5 deg) at 100% of intensity. Then, we measure the errors relative to the Stokes reference input light for each scenario and tested angles.

Table 4 examines the PSNR in terms of the Stokes vector parameters, compiling the mean values and standard deviation (square brackets) evaluated in the above scenarios. The means have different levels in the vector components and they have a peak in the nine angles setup in both S_0 and S_2 components. However, it can be seen that the standard deviation is also higher at this point which counteracts its ranking. Conversely, the use of four angles yields only 2% less in S_0 but with a lowest standard deviation, along with the better values in S_1 and the lowest

Table 4 PSNR mean values and standard deviation (square brackets) in terms of S_0 , S_1 , and S_2 parameters over testing data ranging from 0 deg to 175 deg with a step of 5 deg as a function of training data calculated out of different number of angles (36, 18, 9, 5, 4 and 3 angles) and six intensities levels. Best values are highlighted in bold and worst values in italics.

Training Data	S_0	S_1	S_2
	Mean [Std]	Mean [Std]	Mean [Std]
36 angles	57.46 [5.36]	54.35 [5.59]	55.85 [6.10]
18 angles	57.91 [5.47]	54.44 [5.04]	55.67 [6.06]
9 angles	58.69 [5.48]	54.49 [5.04]	56.28 [5.35]
5 angles	57.83 [4.41]	54.05 [4.79]	54.78 [5.13]
4 angles	57.66 [4.28]	55.03 [3.97]	56.12 [4.40]
3 angles	55.72 [5.09]	52.62 [5.56]	53.55 [5.59]

standard deviation in S_2 . Figure 3 breaks down the said PSNR behavior when sweeping from 0 deg to 175 deg. It can be seen that the signal level is not uniform with respect to the input polarization angle, nevertheless, the fluctuations roughly keep the same shape among scenarios. In this respect, it can be seen that there is a tradeoff between this simplification and the output error when cutting down the amount of training measurements.

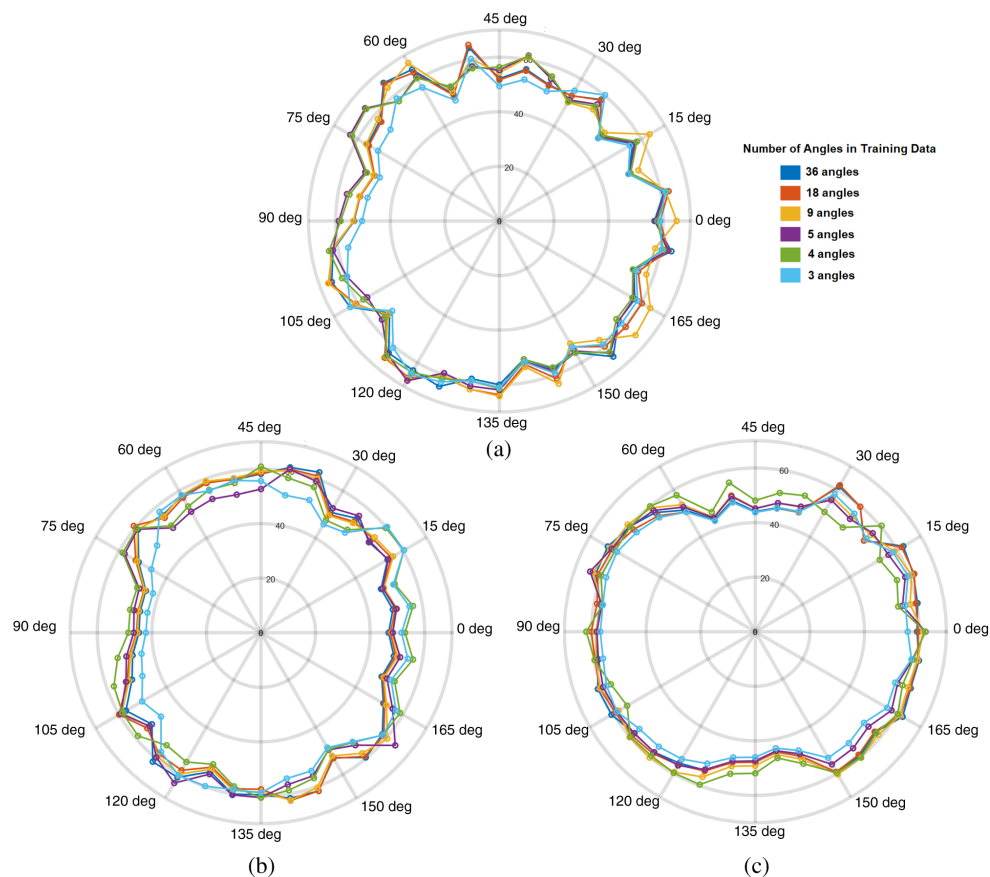


Fig. 3 PSNR of Stokes parameters after calibration as a function of the number of angles used to calculate the training data. Subfigures (a), (b), and (c) depict the signal level in the parameters S_0 , S_1 , and S_2 , respectively. Each color represents a different setup and the radius corresponds to the PSNR. Test images are swept from 0 deg to 175 deg with a step of 5 deg.

Hagen et al.²⁶ discussed a simpler setup to characterize each pixel individually by using measurements in four different angles instead of 54 measurements (nine angles and six intensity levels) as the training data used in the above standalone evaluation. They put forward a single-pixel calibration approach with simpler calibration setup than prior calibration methods.^{16,21,22,27} They use only four measurements to characterize each pixel individually, and recover the incident intensity, the orientation axis of analysis, and the diattenuation parameters. Under this approach, a motorized rotational stage is no longer needed as an angle-graduated mount is a practical solution. This choice is in accordance with our results as we found that it is not necessary to use more than four input angles for the calibration.

By applying the same criteria as in the calibration method comparison in Sec. 4.3, we have verified that characterizing the analysis vectors with four angles with one intensity delivered a 93% RMSE reduction averaged over the three Stokes components respect to the uncalibrated data. Comparatively, the nine angles and six intensity level scenario yields a relatively small improvement (0.67% in RMSE reduction). The difference is attributed to the fact that the latter option considers more than one DR.

6 Discussion and Conclusion

In this paper, we reviewed existing calibration algorithms and evaluated them using a practical implementation applied to a commercially available monochrome PFA camera. The camera's polarization optical parameters were characterized and methods were applied on data using a uniform linearly polarized light. The results obtained show that the primitive superpixel method performs well and that any other methods that are derived from it bring no significant enhancement. We discussed the calibration setup optimization considering the impact of data DR and the impact of training polarization angle selection. This study considers separately the influence of aforementioned two parameters on the signal-to-noise ratio of the Stokes vector components. In this way, it can help arrange the optimal setup when calibrating a PFA camera for a specific application. Our results show that using more than four angle realizations does not significantly improve the PSNR in the Stokes vector components. But we also demonstrate that using two different DR images improves significantly the calibration compared to using one as in Hagen et al.²⁶ To summarize four angles and two DR realization is a good compromise to obtain a good PSNR while simplifying the calibration setup described in Powell and Gruev.¹⁶

Future works would be to define a complete PFA camera pipeline, including high DR enhancement based on multiple exposure times.²⁸ Other PFA sensors came on the market, like the IMX250 MYR from Sony that captures both color and polarization information. In the presence of low and high polarization signatures in the same scene, high DR could correct for saturation and/or nonuniformity of camera sensitivities among all spectral and polarization channels. Finally, the study of polarimetric parameters calibration in multispectral polarimeters, which, in turn, require spectral calibration, is missing in the literature.

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